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SET-Y

PHD-EE-2023-24

Statistics

10009

Sr. No. ....

Time : **1¼ Hours** Max. Marks : **100** Total Questions : **100**

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SEAL

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1. The mean of Binomial distribution $B(n, p)$ is :

(1) np (2) $np - 1$ (3) $np - 2$ (4) $np - 3$

2. Given that $E[X + 4] = 10$ and $E[X + 4]^2 = 116$, then Var[X] is equal to :

(1) 4 (2) 8 (3) 12 (4) 16

3. Let 'X' be a continuous random variable with Probability Density Function

$$f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ -ax + 3a, & 2 \le x \le 3 \\ 0, & x > 3 \end{cases}$$

Then the value of 'a' is given by :

(1) 0.4 (2) 0.5 (3) 0.3 (4) 0.1

4. Let 'X' be a continuous random variable with Probability Density Function

$$f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ -ax + 3a, & 2 \le x \le 3 \\ 0, & x > 3 \end{cases}$$

Then the value of $P(X \le 1.5)$ is given by :

(1) 0.5 (2) 0.81 (3) 0.19 (4) 0.17

5. The moment generating function of Poisson Distribution is :

(1) $e^{\lambda}(e^t - 1)$ (2) $e^t(e^{\lambda} - 1)$ (3) $e^t - 1$ (4) $e^{\lambda}(1 - e^t)$

6. Which of the following is the median of the exponential distribution with parameter λ ?

(1) λ (2) $-\lambda^{-1}$ (3) λ^{-1} (4) λ^{-2}

7. The Quartile Deviation of the normal distribution is :

(1) Q. D. = $\frac{2}{4}\sigma$ (2) Q. D. = $\frac{2}{3}\sigma$ (3) Q. D. = $\frac{2}{5}\sigma$ (4) Q. D. = $\frac{3}{4}\sigma$

8. If A and B are two independent events, then $P(\overline{A} \cap \overline{B})$ is equal to :

(1) $P(\overline{A})P(\overline{B})$ (2) $1 - P(A \cup B)$

(3) $[1 - P(A)][1 - P(B)]$ (4) All of the above

9. If event A and event B has occurred and it is known that $P(B) = 1$, the conditional probability $P(A/B)$ is equal to :

(1) $P(A)$ (2) $P(B)$ (3) One (4) Zero

10. The recurrence formula for geometric distribution is given by :

(1) $p(x+i) = qp(x+i)$ (2) $p(x) = qp(x+i)$

(3) $p(x+i) = qp(x)$ (4) $p(x) = p(x+i)$

11. The correct relationship between A.M, G.M., and H.M. is :

(1) A.M.= G.M.= H.M. (2) G.M. ≥ A.M. ≥ H.M.

(3) H.M. ≥ G.M. ≥ A.M. (4) A.M. ≥ G.M. ≥ H.M

12. Average wages of workers of factory are Rs. 550 per month and the standard deviation of wages is 110. The coefficients of variation is :

(1) 30% (2) 15% (3) 500% (4) 20%

13. If the mode of a frequency distribution is 16 and its mean is 16, then the median of the distribution is :

(1) 0 (2) 16 (3) 32 (4) 8

14. If Quartile deviation of a set of observations is given as 6.4 and the value of first quartile is 5. What is the value of the third quartile ?

(1) 12 (2) 15.3 (3) 17.8 (4) 20.2

15. For a leptokurtic frequency curve, the measures of kurtosis is :

(1) 0 (2) −3 (3) less than 1 (4) greater than 3

16. Standard error of the sample correlation coefficient 'r' is based on 'n' paired values is :

(1) $\frac{1+r^2}{\sqrt{n}}$ (2) $\frac{1+r^2}{n}$ (3) $\frac{1-r^2}{\sqrt{n}}$ (4) $\frac{1+r^2}{\sqrt{n-1}}$

17. Given the following set of equations :

$$x_1 + 4x_2 - x_3 = 3$$
$$5x_1 + 2x_2 + 3x_3 = 4$$

The basic feasible solution involving x_1 and x_2 is :

(1) $\left(\frac{5}{9}, \frac{11}{18}, 0\right)$ (2) $\left(\frac{5}{9}, 0, 0\right)$ (3) $\left(0, \frac{11}{18}, 0\right)$ (4) $\left(\frac{2}{9}, 0, \frac{3}{14}\right)$

18. The range of a partial correlation coefficient is :

(1) 0 to ∞ (2) −∞ to ∞ (3) 0 to 1 (4) −1 to 1

19. The term regression was introduced by :

(1) Sir Francis Galton (2) R.A. Fisher
(3) Karl Pearson (4) P.C. Mahalanobis

20. If Regression Equations are :

$$6y = 5x + 90$$
$$15x = 8y + 30$$

And the variance of $x = 4$, then the correlation coefficient between 'x' and 'y' is :

(1) 0.45 (2) 0.67 (3) 0.78 (4) 0.88

21. The pgf of a certain distribution is given as :

$P(s) = 3s^2 - 2s + 6$

What is the mean of this distribution ?

(1) 2 (2) 3 (3) 4 (4) 5

22. The relation between student's -t and F-distribution is :

(1) $F_{1,1} = t_n^2$ (2) $F_{n,1} = t_1^2$ (3) $F_{1,n} = t_\infty^2$ (4) $F_{1,n} = t_n^2$

23. The minimum variance unbiased estimator of θ^2 based on a sample of size 'n' from $N(\theta\ 1)$ is :

(1) $\bar{X}^2 - 1/n$ (2) $\bar{X}^2 + 1/n$
(3) $\sum(X_i - \bar{X}^2)/n$ (4) $\sum(X_i - \bar{X}^2)/(n-1)$

24. The Bayes estimator of a parameter under squared error loss function is :

(1) Posterior mean (2) Posterior median

(3) Posterior mode (4) Posterior variance

25. The decision criteria in SPRT depends on the function of :

(1) Type I error (2) Type II error

(3) Both type I and type II error (4) Neither Type I nor Type II error

26. What is an unbiased estimator of θ for the distribution $f(x, \theta) = \theta e^{-\theta x}, x \geq 0$?

(1) $\frac{(n-1)\overline{X}}{n}$ (2) $\frac{(n-1)}{n\overline{X}}$ (3) $\frac{\overline{X}}{n-1}$ (4) $\frac{1}{(n-1)\overline{X}}$

27. Given a random sample :

$f(x,\theta) = \frac{2}{\alpha^2}(\alpha - x), 0 < x < \alpha$

What is the MLE of α ?

(1) X (2) $2X$ (3) $\frac{X^2}{2}$ (4) $\frac{X}{2}$

28. For a particular hypothesis test, the probabilities of type I and type II errors are respectively, 0.05 and 0.09. The power of the test is :

(1) 0.95 (2) 0.14 (3) 0.86 (4) 0.91

29. A random sample of 100 articles are taken from a batch of 2000 articles shows that the average diameter of the articles is 0.354 and a standard deviation 0.048. What is the 95% confidence interval for the average diameter of the batch ?

(1) (0.2934,0.4235) (2) (0.3448,0.3632)

(3) (0.3021,0.3824) (4) (0.3923,0.4212)

30. Which of the following is ***true*** ?

(1) Unbiased estimator is always efficient.

(2) Consistent estimator is always unbiased.

(3) Unbiased estimator is always consistent.

(4) MLE is always a function of sufficient statistic.

31. In SRSWOR, if $\bar{y} = 50, n = 100, N = 500$, then the estimated population total is :

(1) 250 (2) 500 (3) 25000 (4) 2500

32. Headquarters of Field Operations Division of NSSO are located at :

(1) New Delhi (2) Kolkata (3) Bombay (4) Chennai

33. The variance of stratified sampling mean $\bar{Y}_{st}$ is :

(1) $\sum_{h=1}^{L}\left(\frac{1}{N_h}-\frac{1}{n_h}\right)W_h^2 S_h^2$

(2) $\sum_{h=1}^{L}\left(\frac{1}{n_h}-\frac{1}{N_h}\right)W_h^2 S_h^2$

(3) $\sum_{h=1}^{L}\left(\frac{1}{N_h}-\frac{1}{n_h}\right)S_h^2$

(4) $\sum_{h=1}^{L}\left(\frac{1}{n_h}-\frac{1}{N_h}\right)W_h^2 s_h^2$

34. In simple random sampling, the biased of ratio estimator is unbiased if :

(1) They are independent (2) They are uncorrelated

(3) They are correlated (4) They are dependent

35. Wishart distribution (σ^2, n) follows :

(1) $\sigma^2\chi^2$ distribution (2) $N(0, \sigma^2)$ distribution

(3) $\frac{e^{-n}\sigma^2}{n}$ distribution (4) Beta distribution

36. Hotelling T^2 can be approximated to statistic 'F' with usual notation as test criteria following the inequality for rejection as :

(1) $T^2 > F_{p,n-p;\alpha}f$ or $n > p$

(2) $T^2 < F_{p,n-p;\alpha}f$ or $n > p$

(3) $T^2 > \frac{n-1}{n-p}F_{p,n-p;\alpha}f$ or $n > p$

(4) $T^2 > \frac{n-p}{n-1}F_{p,n-p;\alpha}f$ or $n > p$

37. A measure of association between a discriminant function and a set of dummy variables that define the group membership is known as :

(1) Multivariate Correlation (2) Multicollinearity

(3) Canonical Correlation (4) Biserial Correlation

38. The ratio of number of replications required in CRD and RBD for the same amount of information is :

(1) 3:2 (2) 5:3 (3) 5:4 (4) 3:5

39. In a Randomised Block Design, we always have :

(1) Number of blocks = Number of treatments

(2) Number of blocks > Number of treatments

(3) Number of blocks < Number of treatments

(4) All of the above

40. The additional effect gained due to combined effect of two or more factors is known as :

(1) Main Effect (2) Interaction Effect

(3) Either (1) or (2) (4) Neither (1) or (2)

41. For a standard $n \times n$ Latin Square, how many different Latin squares can be obtained with the same standard ?

(1) $n!\,(n-2)!$ (2) $(n-1)!\,(n-2)!$

(3) $n!\,(n-1)!$ (4) $n!\,(n+1)!$

42. The condition for the time reversal test to hold good with usual notations is :

(1) $P_{01} \times P_{10} = 1$ (2) $P_{10} \times P_{01} = 0$

(3) $P_{01} / P_{10} = 1$ (4) $P_{01} + P_{10} = 1$

43. Weight in Laspeyre's price index number is known as ....

(1) Quantity during the current year

(2) Quantity in the base year

(3) Price during the current year

(4) Price in the base year

44. In India, the collection of vital statistics started for first time in :

(1) 1720 (2) 1886 (3) 1969 (4) 1946

45. Fisher's ideal formula does not satisfy ....

(1) Time Reversal Test (2) Circular Test

(3) Factor Reversal Test (4) Unit Test

46. If l_x is the number of persons living at the age 'x' and 'L_x' the number of persons living in the mid of 'x' and '$(x+1)$' years, then the relation between l_x and L_x is :

(1) $L_x = \frac{1}{2}(l_x + l_{x+1})$ (2) $L_x = \left(\frac{x}{2} + l_x\right)$

(3) $L_x = l_{x+\frac{1}{2}}$ (4) $L_x = l_{x+\frac{3}{2}}$

47. Fertility rates mainly depend on :

(1) Total female population

(2) Total population

(3) Female population of child bearing age

(4) Number of newly born babies

48. If the quantity demanded of a commodity is unresponsive to change in prices, then the demand of that commodity is :

(1) Perfectly Inelastic (2) Elastic

(3) Unit Elastic (4) Inelastic

49. The elasticity for the demand of the durable goods is :

(1) Zero (2) Equal to unity

(3) Greater than unity (4) Less than Unity

50. Consider the following matrix :

$$P = \begin{bmatrix} 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \\ 1/2 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \end{bmatrix}$$

If P is a stochastic matrix, then which of the following is ***not*** true ?

(1) P is Ergodic (2) P is Regular

(3) P is not Regular (4) Both (1) and (3)

51. Service time in queueing theory is usually assumed to follow :

(1) Poisson Distribution (2) Erlang Distribution

(3) Exponential Distribution (4) Normal Distribution

52. For M|M|1 queueing system, the expected number of customers in systems are :

(1) $L_S = \left(\frac{\lambda}{\mu - \lambda}\right)$ (2) $L_S = \left(\frac{\lambda - \mu}{\lambda}\right)$

(3) $L_S = \left(\frac{\mu}{\mu - \lambda}\right)$ (4) $L_S = \left(\frac{\mu - \lambda}{\lambda}\right)$

53. Which of the following relationships is ***not*** true ?

(1) $W_S = W_q + \frac{1}{\mu}$ (2) $L_S = \lambda W_S$

(3) $L_S = L_q + \frac{1}{\lambda}$ (4) $L_q = \lambda W_q$

54. Maximize $Z = 10x_1 + 25x_2$, subject to $0 \le x_1 \le 3, 0 \le x_2 \le 3, x_1 + x_2 \le 5$

(1) 80 at (3, 2) (2) 75 at (0,3) (3) 30 at (3,0) (4) 95 at (2, 3)

55. Which variable is added for the less than or equal to type of constraint ?

(1) Slack (2) Surplus (3) Artificial (4) Basic

56. The convex combination of two points $\bar{x}_1, \bar{x}_2 \in X$ is referred as :

(1) $(1-\lambda)\bar{x}_1 + \lambda\bar{x}_2, 0 \le \lambda \le 1$

(2) $(1-\lambda)\bar{x}_1 + \lambda\bar{x}_2$, λ is any real number

(3) $\bar{x}_1 + \lambda\bar{x}_2, 0 \le \lambda \le 1$

(4) $\lambda\bar{x}_1 + \lambda\bar{x}_2$, λ is any real number

57. The assignment problem is :

(1) non-linear programming problem

(2) dynamic programming problem

(3) integer linear programming problem

(4) integer non-linear programming problem

58. The order of Convergence of Secant Method is :

(1) 2.4 (2) 2 (3) 1.62 (4) 1

59. If $f(0) = 3, f(1) = 5, f(3) = 21$, then the unique polynomials of degree 2 or less using Newton divided difference interpolation will be :

(1) $2x^2 + 2x + 1$ (2) $2x^2 - 3x + 1$

(3) $2x^2 + 3$ (4) $x^2 + 3x - 2$

60. The third difference of a cubic $\Delta^3 y$ function are :

(1) Constant (2) Not constant

(3) Variables (4) None of the above

61. Let 'f' be a non-constant entire function. Which of the following properties is possible for 'f' for each $z \in \mathbb{C}$?

(1) Re f(z) = Im f(z) (2) $|f(z)| < 1$

(3) $Im(z) < 0$ (4) $f(z) \neq 0$

62. Let 'f' and 'g' be mesomorphic function on ($\mathbb{C}$. If 'f' has a zero of order 'k' at $z = a$ and z has a pole of order 'm' at z = 0, then g(f(z)) has :

(1) a zero of order km at $z = a$

(2) a pole of order km at $z = a$

(3) a zero of order $|k - m|$ at $z = a$

(4) a pole of order $|k - m|$ at $z = a$

63. The set of vectors $X_1 = (2, 1, 4)$, $X_2 = (-3, 2, -1)$, $X_3 = (1, -3, -2)$ is :

(1) Linearly dependent (2) Linearly independent

(3) Both of them (4) None of these

64. Let a be 2 × 2 with Det(A) = 1 & Trace(A) = 3, then Trace(A^2) is :

(1) 7 (2) 8 (3) 9 (4) 10

65. If A is (2 × 2) matrix over IR with Det(A + I) = 1 + Det (A), then we can conclude that :

(1) Det(A) = 0 (2) A = 0 (3) Tr(A) = 0 (4) A is non-singular

66. If A is a square matrix, then $A + A'$, AA' and $A'A$ are :

(1) Symmetric (2) Skew Symmetric

(3) Hermitian (4) Skew Hermitian

67. If $\lim_{n\to\infty} a_n^{1/n} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ provided limit on RHS exists. This result is known as :

(1) Cauchy 1st Theorem on Limits (2) Cauchy 2nd Theorem on Limits

(3) Squeeze Principle (4) Leibnitz' Rule

68. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ is :

(1) 1 (2) 0 (3) e (4) 2

69. Consider the sequence $a_n = \left(1+(-1)^n \frac{1}{n}\right)^n$ then :

(1) $\lim_{n\to\infty} \text{Sup}(a_n) = \lim_{n\to\infty} \text{Inf}(a_n) = 1$ (2) $\lim_{n\to\infty} \text{Sup}(a_n) = \lim_{n\to\infty} \text{Inf}(a_n) = e$

(3) $\lim_{n\to\infty} \text{Sup}(a_n) = \lim_{n\to\infty} \text{Inf}(a_n) = \frac{1}{e}$ (4) $\lim_{n\to\infty} \text{Sup}(a_n) = e, \lim_{n\to\infty} \text{Inf}(a_n) = \frac{1}{e}$

70. If X and Y are standard normal variates with correlation coefficient 'ρ' between them, then the correlation coefficient between X^2 and Y^2 is :

(1) $2\rho - 1$ (2) ρ^2 (3) ρ (4) $\sqrt{\rho}$

71. If the random variables X, Y and Z have the means $\mu_x = 5$, $\mu_y = 7$ and $\mu_z = 4$; $\sigma_X^2 = 10$, $\sigma_Y^2 = 14$ and $\sigma_Z^2 = 20$; Cov(X, Y) = 1, Cov (X, Z) = 3 and Cov(Y, Z) = 2, then what is the covariance of $U = X + 4Y + 2Z$ and $V = 3X - Y - Z$?

(1) −76 (2) 82 (3) −82 (4) 76

72. Neelam has appeared in an examination which follows multiple choice questions, each having five possible answers. The probability that she knows an answer is 0.75. If she does not know an answer, she will guess, with the conditional probability 1/5 of being correct. The conditional probability that Neelam knows the answer, given that she gives the correct answer is :

(1) 0.25 (2) 0.80 (3) 0.90 (4) 0.94

73. A drunkard takes a forward step with probability 'p' and a backward step with probability 'q'. After taking 11 steps, the probability that he is the one step away from the starting point is :

(1) $p^6 + q^6$ (2) $2(p^6 + q^5)$ (3) $462\, p^6 q^5$ (4) $462\, p^5 q^5$

74. For the sequence $\{ X_n \}$ of independent random variables the following are defined :

$P(X_k = \pm 2^k) = 2^{-(2k+1)}$; $P(X_k = 0) = 1 - 2^{-2k}$

Which of the following are not correct(s) ?

(a) WLLN holds because $E(X_k) = 0$ and $\lim_{n\to\infty} \frac{B_n}{n^2} \to 0$ where $B_n = \text{Var}\left(\sum_{i=1}^{n} X_i\right)$

(b) Weak law of large numbers holds by Khinchin's theorem because $E(X_k)$ is finite.

(c) WLLN holds since X_k are identically distributed.

Select the correct answer using code given below :

(1) (b) and (c) only (2) (a) and (b) only

(3) (a) and (c) only (4) (a), (b) and (c) only

75. If σ_X^2, σ_Y^2 and σ_{X-Y}^2 are the variances of X, Y and $X - Y$ respectively, then what is the coefficient of correlation between 'X' and 'Y' ?

(1) $\frac{\sigma_X^2 + \sigma_Y^2 - \sigma_{X-Y}^2}{2\sigma_X \sigma_Y}$ (2) $\frac{\sigma_X^2 + \sigma_Y^2}{2\sigma_X \sigma_Y}$

(3) $\frac{\sigma_X^2 + \sigma_Y^2 - \sigma_{X+Y}^2}{2\sigma_X \sigma_Y}$ (4) $\frac{\sigma_X^2 - \sigma_Y^2}{2\sigma_X \sigma_Y}$

76. Let 'X' and 'Y' be independent Gamma $G(\alpha_1, \beta)$ and $G(\alpha_2, \beta)$ random variables respectively. Then $\frac{X}{X+Y}$ is distributed as :

(1) $G(\alpha_1 + \alpha_2, \beta)$ (2) $\beta_1(\alpha_1, \alpha_2)$

(3) $U(0, 1)$ (4) $G(\alpha_1, \alpha_2)$

77. The Joint Density Function of 'X' and 'Y' is given by :

$$f(x,y)=\begin{cases}2e^{-x}e^{-2y} & 0<x<\infty, 0<y<\infty\\ 0 & \text{otherwise}\end{cases}$$

What is the value of P(X < Y) ?

(1) 1/3 (2) 1/2 (3) 1/4 (4) 1/6

78. For a certain frequency distribution, the numerical computation yields the following : Mean = 62, Median = 65 , Coefficient of skewness = −0.3, then the standard deviation is equal to :

(1) 10 (2) 30 (3) 90 (4) 300

79. Let 'X' be a random variable having Probability Density Function :

$$f(x)=\begin{cases}x/2 & 0<x<1\\ 1/2 & 1<x\le 2\\ (3-x)/3 & 2<x\le 3\end{cases}$$

Then P(1.5 < X < 2.5|X > 1) equals to :

(1) 3/8 (2) 5/8 (3) 1/2 (4) 1/4

80. Let $X_1, X_2, \ldots\ldots, X_n$ be i.i.d. random variables with $E(X_i)=\mu$ and $E(X_i^2)<\infty$, then the consistent estimator for μ is :

(1) $\dfrac{2i^2}{n(n+1)\sum X_i}$ (2) $\dfrac{2}{n(n+1)}\sum iX_i$

(3) $\dfrac{2i}{n(n+1)}\sum X_i$ (4) $\dfrac{2}{n(n+1)}\sum i^2 X_i$

81. Let Y_1 , Y_2 , Y_3 be uncorrelated observations with common variance σ^2 and expectations given by $(Y_1)=\beta_1$, $E(Y_2)=\beta_2$ and $E(Y_3)=\beta_1+\beta_2$ where β_1 and β_2 are unknown parameters. The best linear unbiased estimator of $\beta_1+\beta_2$ is :

(1) $\frac{1}{2}(Y_1+Y_2+Y_3)$ (2) Y_1+Y_2

(3) $\frac{1}{2}(Y_1+Y_2+2Y_3)$ (4) $\frac{1}{2}(Y_1-Y_2-Y_3)$

82. Under the regulatory conditions, if λ_n is the likelihood ratio, then the asymptotic distribution of $-2\log\lambda_n$ as $n \to \infty$ is :

(1) Normal distribution
(2) Chi square distribution
(3) F-distribution
(4) T distribution

83. If the pdf of a random variable 'X' is $f(x, \theta) = 1 - \theta^2$, $0 \le x \le \frac{1}{1-\theta^2}$ and if we reject $H_0 : \theta = \frac{1}{2}$ against the alternative $H_1 : \theta = \frac{3}{4}$ whenever $x \ge 1$, then what is the power of the test ?

(1) $\frac{15}{16}$
(2) $\frac{1}{2}$
(3) $\frac{7}{16}$
(4) $\frac{9}{16}$

84. If $X_1, X_2, \ldots\ldots X_n$ is a random sample form poisson distribution with parameter 'λ', then the maximum likelihood estimator of log λ is given by :

(1) $e^{\left(\Sigma_{i=1}^{n} \frac{X_i}{n}\right)}$
(2) $\log\left(\sum_{i=1}^{n} \frac{X_i}{n}\right)$
(3) $e^{\left(\Sigma_{i=1}^{n} X_i\right)}$
(4) $\log\left(\sum_{i=1}^{n} X_i\right)$

85. Which of the following could be used as a test for autocorrelation up to third order ?

(1) Spearmen Correlation test
(2) The Breusch-Goldfrey test
(3) The Durbin Watson test
(4) The Gold-Fled Quandt test

86. The following equation represent a simultaneous equation model :

$$K_1 = \alpha_1 K_2 + \beta_1 Z_1 + u_1$$

$$K_2 = \alpha_2 K_1 + \beta_2 Z_2 + u_2$$

OLS will suffer from simultaneous bias if :

(1) u_1 is correlated with Z_1
(2) Z_1 is correlated with Z_2
(3) K_2 is correlated with u_1
(4) K_1 is correlated with u_1

87. Having known the last census population 'P_0' and growth rate 'r', the population after 'n' years based on compound interest formula will be :

(1) $\hat{P}_t = P_0(1+r)^n$
(2) $\hat{P}_t = P_0(1+n)^r$
(3) $\hat{P}_t = P_0/(1+r)^n$
(4) $\hat{P}_t = P_0/(1+n)^r$

88. Vital rates are customarily expressed as :

(1) Percentages (2) Per Thousand (3) Per Million (4) Per Trillion

89. Age - Specific mortality rates fail to reveal :

(1) Mortality conditions
(2) Age-distribution of persons
(3) Sex Ratio
(4) All of the above

90. The relationship between NRR and GRR is :

(1) NRR and GRR are usually equal

(2) NRR can never exceed GRR

(3) NRR is generally greater than GRR

(4) None of the above

91. Construction of life tables is based on the assumption that :

(1) Age - specific death rates are constant at all ages.

(2) Death rates are uniformly distributed between two birth days.

(3) Mortality rates are same for male and female populations.

(4) All of the above

92. A life table consists of :

(1) Seven columns
(2) Eight columns
(3) Six columns
(4) Nine Columns

93. King's abridged life tables are based in the calculation of :

(1) Central mortality rate

(2) The number of persons and deaths for central age in the interval $\{x, x + n\}$

(3) Both (1) and (2)

(4) Neither (1) nor (2)

94. If Laspeyre's price index number is 324 and Paasche's Price Index number is 144, then Fisher's Ideal Index Number is :

(1) 180 (2) 234 (3) 216 (4) 222

95. If a negative value appears in the solution values (X_B) column of the simplex table, then :

(1) The solution is optimal (2) The solution is infeasible

(3) The solution is unbounded (4) All of the above

96. The maximum number of extreme points for a LPP

Max $Z = cx$

Subject to

$Ax = b; x \geq 0$

Where A is $m \times n$ matrix is equal to :

(1) $\frac{m!}{n!(m-n)!}$ (2) $\frac{n!}{m!(n-m)!}$ (3) $(m - n)$ (4) mn

97. In a basic feasible solution of an $m \times n$ transportation problem, the number of positive allocations is atmost :

(1) $m + n$ (2) $m + n - 1$ (3) $m - n$ (4) $m + n - 2$

98. The necessary and sufficient condition for the existence of a feasible solution of a transportation problem is :

(1) $\sum a_i = \sum b_j$ (2) $\sum a_i \neq \sum b_j$ (3) $\sum a_i = 0$ (4) $\sum b_j = 0$

99. Consider the linear programming problem :

Max $Z = 3x_1 + 2x_2$

Subject to

$$x_1 + x_2 \le 4$$

$$x_1 - x_2 \le 2$$

$$x_1, x_2 \ge 0$$

Then its solution is :

(1) $x_1 = 3,\ x_2 = 1$, Max $Z = 11$ (2) $x_1 = 1,\ x_2 = 3$, Max $Z = 10$

(3) $x_1 = 2,\ x_2 = 1$, Max $Z = 11$ (4) $x_1 = 1,\ x_2 = 2$, Max $Z = 10$

100. The probability of living of a person in the age group 'x' to '$(x + n)$' can be obtained by the formula :

(1) l_{x+n} / l_x (2) $(l_x - l_{x+n}) / l_{x+n}$

(3) $(l_x - l_{x+n}) / l_x$ (4) l_x / l_{x+n}

Total No. of Printed Pages : 17

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B

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Max. Marks : 100

Total Questions : 100

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Date of Examination____________

(Signature of the Candidate)

(Signature of the Invigilator)

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2. The candidates ***must return*** the question booklet as well as OMR Answer-Sheet to the Invigilator concerned before leaving the Examination Hall, failing which a case of use of unfair-means / mis-behaviour will be registered against him / her, in addition to lodging of an FIR with the police. Further the answer-sheet of such a candidate will not be evaluated.
3. Keeping in view the transparency of the examination system, carbonless OMR Sheet is provided to the candidate so that a copy of OMR Sheet may be kept by the candidate.
4. Question Booklet along with answer key of all the A, B, C & D code shall be got uploaded on the University Website immediately after the conduct of Entrance Examination. Candidates may raise valid objection/complaint if any, with regard to discrepancy in the question booklet/answer key within 24 hours of uploading the same on the University Website. The complaint be sent by the students to the Controller of Examinations by hand or through email. Thereafter, no complaint in any case, will be considered.
5. The candidate ***must not*** do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question booklet itself. Answers ***must not*** be ticked in the question booklet.
6. ***There will be no negative marking. Each correct answer will be awarded one full mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.***
7. Use only **Black** or **Blue Ball Point Pen** of good quality in the OMR Answer-Sheet.
8. *Before answering the questions, the candidates should ensure that they have been supplied correct and complete booklet. Complaints, if any, regarding misprinting etc. will not be entertained 30 minutes after starting of the examination.*

1. For a standard $n \times n$ Latin Square, how many different Latin squares can be obtained with the same standard ?
   (1) $n!\,(n-2)!$
   (2) $(n-1)!\,(n-2)!$
   (3) $n!\,(n-1)!$
   (4) $n!\,(n+1)!$

2. The condition for the time reversal test to hold good with usual notations is :
   (1) $P_{01} \times P_{10} = 1$
   (2) $P_{10} \times P_{01} = 0$
   (3) $P_{01} / P_{10} = 1$
   (4) $P_{01} + P_{10} = 1$

3. Weight in Laspeyre's price index number is known as ....
   (1) Quantity during the current year
   (2) Quantity in the base year
   (3) Price during the current year
   (4) Price in the base year

4. In India, the collection of vital statistics started for first time in :
   (1) 1720
   (2) 1886
   (3) 1969
   (4) 1946

5. Fisher's ideal formula does not satisfy ....
   (1) Time Reversal Test
   (2) Circular Test
   (3) Factor Reversal Test
   (4) Unit Test

6. If l_x is the number of persons living at the age 'x' and 'L_x' the number of persons living in the mid of 'x' and '$(x+1)$' years, then the relation between l_x and L_x is :
   (1) $L_x = \frac{1}{2}(l_x + l_{x+1})$
   (2) $L_x = \left(\frac{x}{2} + l_x\right)$
   (3) $L_x = l_{x+\frac{1}{2}}$
   (4) $L_x = l_{x+\frac{3}{2}}$

7. Fertility rates mainly depend on :
   (1) Total female population
   (2) Total population
   (3) Female population of child bearing age
   (4) Number of newly born babies

8. If the quantity demanded of a commodity is unresponsive to change in prices, then the demand of that commodity is :

(1) Perfectly Inelastic (2) Elastic

(3) Unit Elastic (4) Inelastic

9. The elasticity for the demand of the durable goods is :

(1) Zero (2) Equal to unity

(3) Greater than unity (4) Less than Unity

10. Consider the following matrix :

$$P = \begin{bmatrix} 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \\ 1/2 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \end{bmatrix}$$

If P is a stochastic matrix, then which of the following is ***not*** true ?

(1) P is Ergodic (2) P is Regular

(3) P is not Regular (4) Both (1) and (3)

11. If the random variables X, Y and Z have the means $\mu_x = 5$, $\mu_y = 7$ and $\mu_z = 4$; $\sigma_X^2 = 10$, $\sigma_Y^2 = 14$ and $\sigma_Z^2 = 20$; Cov(X, Y) = 1, Cov (X, Z) = 3 and Cov(Y, Z) = 2, then what is the covariance of $U = X + 4Y + 2Z$ and $V = 3X - Y - Z$?

(1) –76 (2) 82 (3) –82 (4) 76

12. Neelam has appeared in an examination which follows multiple choice questions, each having five possible answers. The probability that she knows an answer is 0.75. If she does not know an answer, she will guess, with the conditional probability 1/5 of being correct. The conditional probability that Neelam knows the answer, given that she gives the correct answer is :

(1) 0.25 (2) 0.80 (3) 0.90 (4) 0.94

13. A drunkard takes a forward step with probability 'p' and a backward step with probability 'q'. After taking 11 steps, the probability that he is the one step away from the starting point is :

(1) $p^6 + q^6$ (2) $2(p^6 + q^5)$

(3) $462\, p^6 q^5$ (4) $462\, p^5 q^5$

14. For the sequence $\{ X_n \}$ of independent random variables the following are defined :

$P(X_k = \pm 2^k) = 2^{-(2k+1)}$; $P(X_k = 0) = 1 - 2^{-2k}$

Which of the following are not correct(s) ?

(a) WLLN holds because $E(X_k) = 0$ and $\lim_{n\to\infty} \frac{B_n}{n^2} \to 0$ where $B_n = \text{Var}\left(\Sigma_{i=1}^{n} X_i\right)$

(b) Weak law of large numbers holds by Khinchin's theorem because $E(X_k)$ is finite.

(c) WLLN holds since X_k are identically distributed.

Select the correct answer using code given below :

(1) (b) and (c) only
(2) (a) and (b) only
(3) (a) and (c) only
(4) (a), (b) and (c) only

15. If σ_X^2, σ_Y^2 and σ_{X-Y}^2 are the variances of X, Y and $X - Y$ respectively, then what is the coefficient of correlation between 'X' and 'Y' ?

(1) $\frac{\sigma_X^2 + \sigma_Y^2 - \sigma_{X-Y}^2}{2\sigma_X\sigma_Y}$
(2) $\frac{\sigma_X^2 + \sigma_Y^2}{2\sigma_X\sigma_Y}$
(3) $\frac{\sigma_X^2 + \sigma_Y^2 - \sigma_{X+Y}^2}{2\sigma_X\sigma_Y}$
(4) $\frac{\sigma_X^2 - \sigma_Y^2}{2\sigma_X\sigma_Y}$

16. Let 'X' and 'Y' be independent Gamma $G(\alpha_1, \beta)$ and $G(\alpha_2, \beta)$ random variables respectively. Then $\frac{X}{X+Y}$ is distributed as :

(1) $G(\alpha_1 + \alpha_2, \beta)$
(2) $\beta_1(\alpha_1, \alpha_2)$
(3) $U(0, 1)$
(4) $G(\alpha_1, \alpha_2)$

17. The Joint Density Function of 'X' and 'Y' is given by :

$$f(x,y) = \begin{cases} 2e^{-x}e^{-2y} & 0 < x < \infty, 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

What is the value of P(X < Y) ?

(1) 1/3
(2) 1/2
(3) 1/4
(4) 1/6

18. For a certain frequency distribution, the numerical computation yields the following :
Mean = 62, Median = 65 , Coefficient of skewness = –0.3, then the standard deviation is equal to :

(1) 10 (2) 30 (3) 90 (4) 300

19. Let 'X' be a random variable having Probability Density Function :

$$f(x)=\begin{cases} x/2 & 0<x<1 \\ 1/2 & 1<x\le 2 \\ (3-x)/3 & 2<x\le 3 \end{cases}$$

Then $P(1.5 < X < 2.5 | X > 1)$ equals to :

(1) 3/8 (2) 5/8 (3) 1/2 (4) 1/4

20. Let $X_1, X_2, \ldots\ldots X_n$ be i.i.d. random variables with $E(X_i) = \mu$ and $E(X_i^2) < \infty$, then the consistent estimator for μ is :

(1) $\frac{2i^2}{n(n+1)\sum X_i}$

(2) $\frac{2}{n(n+1)}\sum iX_i$

(3) $\frac{2i}{n(n+1)}\sum X_i$

(4) $\frac{2}{n(n+1)}\sum i^2 X_i$

21. Construction of life tables is based on the assumption that :

(1) Age - specific death rates are constant at all ages.

(2) Death rates are uniformly distributed between two birth days.

(3) Mortality rates are same for male and female populations.

(4) All of the above

22. A life table consists of :

(1) Seven columns (2) Eight columns

(3) Six columns (4) Nine Columns

23. King's abridged life tables are based in the calculation of :

(1) Central mortality rate

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(3) Both (1) and (2)

(4) Neither (1) nor (2)

24. If Laspeyre's price index number is 324 and Paasche's Price Index number is 144, then Fisher's Ideal Index Number is :

(1) 180 (2) 234 (3) 216 (4) 222

25. If a negative value appears in the solution values (X_B) column of the simplex table, then :

(1) The solution is optimal (2) The solution is infeasible

(3) The solution is unbounded (4) All of the above

26. The maximum number of extreme points for a LPP

Max $Z = cx$

Subject to

$Ax = b; x \geq 0$

Where A is $m \times n$ matrix is equal to :

(1) $\frac{m!}{n!(m-n)!}$ (2) $\frac{n!}{m!(n-m)!}$ (3) $(m-n)$ (4) mn

27. In a basic feasible solution of an $m \times n$ transportation problem, the number of positive allocations is atmost :

(1) $m+n$ (2) $m+n-1$ (3) $m-n$ (4) $m+n-2$

28. The necessary and sufficient condition for the existence of a feasible solution of a transportation problem is :

(1) $\sum a_i = \sum b_j$ (2) $\sum a_i \neq \sum b_j$ (3) $\sum a_i = 0$ (4) $\sum b_j = 0$

29. Consider the linear programming problem :

Max $Z = 3x_1 + 2x_2$

Subject to

$$x_1 + x_2 \le 4$$

$$x_1 - x_2 \le 2$$

$$x_1, x_2 \ge 0$$

Then its solution is :

(1) $x_1 = 3,\ x_2 = 1$, Max $Z = 11$

(2) $x_1 = 1,\ x_2 = 3$, Max $Z = 10$

(3) $x_1 = 2,\ x_2 = 1$, Max $Z = 11$

(4) $x_1 = 1,\ x_2 = 2$, Max $Z = 10$

30. The probability of living of a person in the age group 'x' to '$(x + n)$' can be obtained by the formula :

(1) l_{x+n} / l_x

(2) $(l_x - l_{x+n}) / l_{x+n}$

(3) $(l_x - l_{x+n}) / l_x$

(4) l_x / l_{x+n}

31. The mean of Binomial distribution $B(n, p)$ is :

(1) np

(2) $np - 1$

(3) $np - 2$

(4) $np - 3$

32. Given that $E[X + 4] = 10$ and $E[X + 4]^2 = 116$, then Var$[X]$ is equal to :

(1) 4

(2) 8

(3) 12

(4) 16

33. Let 'X' be a continuous random variable with Probability Density Function

$$f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ -ax + 3a, & 2 \le x \le 3 \\ 0, & x > 3 \end{cases}$$

Then the value of 'a' is given by :

(1) 0.4

(2) 0.5

(3) 0.3

(4) 0.1

34. Let 'X' be a continuous random variable with Probability Density Function

$$f(x)=\begin{cases} ax, & 0\le x\le 1 \\ a, & 1\le x\le 2 \\ -ax+3a, & 2\le x\le 3 \\ 0, & x>3 \end{cases}$$

Then the value of $P(X \le 1.5)$ is given by :

(1) 0.5 (2) 0.81 (3) 0.19 (4) 0.17

35. The moment generating function of Poisson Distribution is :

(1) $e^{\lambda}(e^{t}-1)$ (2) $e^{t}(e^{\lambda}-1)$ (3) $e^{t}-1$ (4) $e^{\lambda}(1-e^{t})$

36. Which of the following is the median of the exponential distribution with parameter λ ?

(1) λ (2) $-\lambda^{-1}$ (3) λ^{-1} (4) λ^{-2}

37. The Quartile Deviation of the normal distribution is :

(1) Q. D. = $\frac{2}{4}\sigma$ (2) Q. D. = $\frac{2}{3}\sigma$ (3) Q. D. = $\frac{2}{5}\sigma$ (4) Q. D. = $\frac{3}{4}\sigma$

38. If A and B are two independent events, then $P(\overline{A} \cap \overline{B})$ is equal to :

(1) $P(\overline{A})P(\overline{B})$ (2) $1 - P(A \cup B)$

(3) $[1 - P(A)][1 - P(B)]$ (4) All of the above

39. If event A and event B has occurred and it is known that $P(B) = 1$, the conditional probability $P(A/B)$ is equal to :

(1) $P(A)$ (2) $P(B)$ (3) One (4) Zero

40. The recurrence formula for geometric distribution is given by :

(1) $p(x + i) = qp(x + i)$ (2) $p(x) = qp(x + i)$

(3) $p(x + i) = qp(x)$ (4) $p(x) = p(x + i)$

41. Service time in queueing theory is usually assumed to follow :

(1) Poisson Distribution (2) Erlang Distribution

(3) Exponential Distribution (4) Normal Distribution

42. For M|M|1 queueing system, the expected number of customers in systems are :

(1) $L_S = \left(\frac{\lambda}{\mu - \lambda}\right)$

(2) $L_S = \left(\frac{\lambda - \mu}{\lambda}\right)$

(3) $L_S = \left(\frac{\mu}{\mu - \lambda}\right)$

(4) $L_S = \left(\frac{\mu - \lambda}{\lambda}\right)$

43. Which of the following relationships is ***not*** true ?

(1) $W_S = W_q + \frac{1}{\mu}$

(2) $L_S = \lambda W_S$

(3) $L_S = L_q + \frac{1}{\lambda}$

(4) $L_q = \lambda W_q$

44. Maximize $Z = 10x_1 + 25x_2$, subject to $0 \le x_1 \le 3, 0 \le x_2 \le 3, x_1 + x_2 \le 5$

(1) 80 at (3, 2) (2) 75 at (0,3) (3) 30 at (3,0) (4) 95 at (2, 3)

45. Which variable is added for the less than or equal to type of constraint ?

(1) Slack (2) Surplus (3) Artificial (4) Basic

46. The convex combination of two points $\bar{x}_1, \bar{x}_2 \in X$ is referred as :

(1) $(1-\lambda)\bar{x}_1 + \lambda\bar{x}_2, 0 \le \lambda \le 1$

(2) $(1-\lambda)\bar{x}_1 + \lambda\bar{x}_2$, λ is any real number

(3) $\bar{x}_1 + \lambda\bar{x}_2, 0 \le \lambda \le 1$

(4) $\lambda\bar{x}_1 + \lambda\bar{x}_2$, λ is any real number

47. The assignment problem is :

(1) non-linear programming problem

(2) dynamic programming problem

(3) integer linear programming problem

(4) integer non-linear programming problem

48. The order of Convergence of Secant Method is :

(1) 2.4 (2) 2 (3) 1.62 (4) 1

49. If $f(0) = 3, f(1) = 5, f(3) = 21$, then the unique polynomials of degree 2 or less using Newton divided difference interpolation will be :

(1) $2x^2 + 2x + 1$ (2) $2x^2 - 3x + 1$

(3) $2x^2 + 3$ (4) $x^2 + 3x - 2$

50. The third difference of a cubic $\Delta^3 y$ function are :

(1) Constant (2) Not constant

(3) Variables (4) None of the above

51. Let 'f' be a non-constant entire function. Which of the following properties is possible for 'f' for each $z \in \mathbb{C}$?

(1) Re f(z) = Im f(z) (2) $|f(z)| < 1$

(3) $Im(z) < 0$ (4) $f(z) \neq 0$

52. Let 'f' and 'g' be mesomorphic function on ($\mathbb{C}$. If 'f' has a zero of order 'k' at $z = a$ and z has a pole of order 'm' at z = 0, then g(f(z)) has :

(1) a zero of order km at $z = a$

(2) a pole of order km at $z = a$

(3) a zero of order $|k - m|$ at $z = a$

(4) a pole of order $|k - m|$ at $z = a$

53. The set of vectors $X_1 = (2, 1, 4)$, $X_2 = (-3, 2, -1)$, $X_3 = (1, -3, -2)$ is :

(1) Linearly dependent (2) Linearly independent

(3) Both of them (4) None of these

54. Let a be 2×2 with Det(A) = 1 & Trace(A) = 3, then Trace(A^2) is :

(1) 7 (2) 8 (3) 9 (4) 10

55. If A is (2×2) matrix over IR with Det(A + I) = 1 + Det (A), then we can conclude that :

(1) Det(A) = 0 (2) A = 0

(3) Tr(A) = 0 (4) A is non-singular

56. If A is a square matrix, then $A + A'$, AA' and $A'A$ are :

(1) Symmetric
(2) Skew Symmetric
(3) Hermitian
(4) Skew Hermitian

57. If $\lim_{n\to\infty} a_n^{1/n} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ provided limit on RHS exists. This result is known as :

(1) Cauchy 1st Theorem on Limits
(2) Cauchy 2nd Theorem on Limits
(3) Squeeze Principle
(4) Leibnitz' Rule

58. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ is :

(1) 1
(2) 0
(3) e
(4) 2

59. Consider the sequence $a_n = \left(1+(-1)^n \frac{1}{n}\right)^n$ then :

(1) $\lim_{n\to\infty} \text{Sup}(a_n) = \lim_{n\to\infty} \text{Inf}(a_n) = 1$
(2) $\lim_{n\to\infty} \text{Sup}(a_n) = \lim_{n\to\infty} \text{Inf}(a_n) = e$
(3) $\lim_{n\to\infty} \text{Sup}(a_n) = \lim_{n\to\infty} \text{Inf}(a_n) = \frac{1}{e}$
(4) $\lim_{n\to\infty} \text{Sup}(a_n) = e$, $\lim_{n\to\infty} \text{Inf}(a_n) = \frac{1}{e}$

60. If X and Y are standard normal variates with correlation coefficient 'ρ' between them, then the correlation coefficient between X^2 and Y^2 is :

(1) $2\rho - 1$
(2) ρ^2
(3) ρ
(4) $\sqrt{\rho}$

61. The pgf of a certain distribution is given as :
$P(s) = 3s^2 - 2s + 6$
What is the mean of this distribution ?

(1) 2
(2) 3
(3) 4
(4) 5

62. The relation between student's -t and F-distribution is :

(1) $F_{1,1} = t_n^2$ (2) $F_{n,1} = t_1^2$ (3) $F_{1,n} = t_\infty^2$ (4) $F_{1,n} = t_n^2$

63. The minimum variance unbiased estimator of θ^2 based on a sample of size 'n' from $N(\theta\ 1)$ is :

(1) $\overline{X}^2 - 1/n$ (2) $\overline{X}^2 + 1/n$

(3) $\sum(X_i - \overline{X}^2)/n$ (4) $\sum(X_i - \overline{X}^2)/(n-1)$

64. The Bayes estimator of a parameter under squared error loss function is :

(1) Posterior mean (2) Posterior median

(3) Posterior mode (4) Posterior variance

65. The decision criteria in SPRT depends on the function of :

(1) Type I error

(2) Type II error

(3) Both type I and type II error

(4) Neither Type I nor Type II error

66. What is an unbiased estimator of θ for the distribution $f(x, \theta) = \theta e^{-\theta} x, x \geq 0$?

(1) $\frac{(n-1)\overline{X}}{n}$ (2) $\frac{(n-1)}{n\overline{X}}$

(3) $\frac{\overline{X}}{n-1}$ (4) $\frac{1}{(n-1)\overline{X}}$

67. Given a random sample :

$f(x,\theta) = \frac{2}{\alpha^2}(\alpha - x), 0 < x < \alpha$

What is the MLE of α ?

(1) X (2) $2X$ (3) $\frac{X^2}{2}$ (4) $\frac{X}{2}$

68. For a particular hypothesis test, the probabilities of type I and type II errors are respectively, 0.05 and 0.09. The power of the test is :

(1) 0.95 (2) 0.14 (3) 0.86 (4) 0.91

69. A random sample of 100 articles are taken from a batch of 2000 articles shows that the average diameter of the articles is 0.354 and a standard deviation 0.048. What is the 95% confidence interval for the average diameter of the batch ?

(1) (0.2934,0.4235)
(2) (0.3448,0.3632)
(3) (0.3021,0.3824)
(4) (0.3923,0.4212)

70. Which of the following is ***true*** ?

(1) Unbiased estimator is always efficient.
(2) Consistent estimator is always unbiased.
(3) Unbiased estimator is always consistent.
(4) MLE is always a function of sufficient statistic.

71. The correct relationship between A.M, G.M., and H.M. is :

(1) A.M.= G.M.= H.M.
(2) G.M. ≥ A.M. ≥ H.M.
(3) H.M. ≥ G.M. ≥ A.M.
(4) A.M. ≥ G.M. ≥ H.M

72. Average wages of workers of factory are Rs. 550 per month and the standard deviation of wages is 110. The coefficients of variation is :

(1) 30%
(2) 15%
(3) 500%
(4) 20%

73. If the mode of a frequency distribution is 16 and its mean is 16, then the median of the distribution is :

(1) 0
(2) 16
(3) 32
(4) 8

74. If Quartile deviation of a set of observations is given as 6.4 and the value of first quartile is 5. What is the value of the third quartile ?

(1) 12
(2) 15.3
(3) 17.8
(4) 20.2

75. For a leptokurtic frequency curve, the measures of kurtosis is :

(1) 0
(2) −3
(3) less than 1
(4) greater than 3

76. Standard error of the sample correlation coefficient 'r' is based on 'n' paired values is :

(1) $\frac{1+r^2}{\sqrt{n}}$
(2) $\frac{1+r^2}{n}$
(3) $\frac{1-r^2}{\sqrt{n}}$
(4) $\frac{1+r^2}{\sqrt{n-1}}$

77. Given the following set of equations :

$$x_1 + 4x_2 - x_3 = 3$$
$$5x_1 + 2x_2 + 3x_3 = 4$$

The basic feasible solution involving x_1 and x_2 is :

(1) $\left(\frac{5}{9}, \frac{11}{18}, 0\right)$ (2) $\left(\frac{5}{9}, 0, 0\right)$ (3) $\left(0, \frac{11}{18}, 0\right)$ (4) $\left(\frac{2}{9}, 0, \frac{3}{14}\right)$

78. The range of a partial correlation coefficient is :

(1) 0 to ∞ (2) $-\infty$ to ∞ (3) 0 to 1 (4) -1 to 1

79. The term regression was introduced by :

(1) Sir Francis Galton (2) R.A. Fisher

(3) Karl Pearson (4) P.C. Mahalanobis

80. If Regression Equations are :

$$6y = 5x + 90$$
$$15x = 8y + 30$$

And the variance of $x = 4$, then the correlation coefficient between 'x' and 'y' is :

(1) 0.45 (2) 0.67 (3) 0.78 (4) 0.88

81. Let Y_1 , Y_2 , Y_3 be uncorrelated observations with common variance σ^2 and expectations given by $(Y_1) = \beta_1$, $E(Y_2) = \beta_2$ and $E(Y_3) = \beta_1 + \beta_2$ where β_1 and β_2 are unknown parameters. The best linear unbiased estimator of $\beta_1 + \beta_2$ is :

(1) $\frac{1}{2}(Y_1 + Y_2 + Y_3)$ (2) $Y_1 + Y_2$

(3) $\frac{1}{2}(Y_1 + Y_2 + 2Y_3)$ (4) $\frac{1}{2}(Y_1 - Y_2 - Y_3)$

82. Under the regulatory conditions, if λ_n is the likelihood ratio, then the asymptotic distribution of $-2\log\lambda_n$ as $n \to \infty$ is :

(1) Normal distribution (2) Chi square distribution

(3) F-distribution (4) T distribution

83. If the pdf of a random variable 'X' is $f(x, \theta) = 1 - \theta^2, 0 \leq x \leq \frac{1}{1-\theta^2}$ and if we reject $H_0 : \theta = \frac{1}{2}$ against the alternative $H_1 : \theta = \frac{3}{4}$ whenever $x \geq 1$, then what is the power of the test ?

(1) $\frac{15}{16}$ (2) $\frac{1}{2}$ (3) $\frac{7}{16}$ (4) $\frac{9}{16}$

84. If $X_1, X_2, \ldots\ldots X_n$ is a random sample form poisson distribution with parameter 'λ', then the maximum likelihood estimator of log λ is given by :

(1) $e^{\left(\Sigma_{i=1}^{n} \frac{X_i}{n}\right)}$ (2) $\log\left(\sum_{i=1}^{n} \frac{X_i}{n}\right)$ (3) $e^{\left(\Sigma_{i=1}^{n} X_i\right)}$ (4) $\log\left(\sum_{i=1}^{n} X_i\right)$

85. Which of the following could be used as a test for autocorrelation up to third order ?

(1) Spearmen Correlation test (2) The Breusch-Goldfrey test

(3) The Durbin Watson test (4) The Gold-Fled Quandt test

86. The following equation represent a simultaneous equation model :

$$K_1 = \alpha_1 K_2 + \beta_1 Z_1 + u_1$$

$$K_2 = \alpha_2 K_1 + \beta_2 Z_2 + u_2$$

OLS will suffer from simultaneous bias if :

(1) u_1 is correlated with Z_1 (2) Z_1 is correlated with Z_2

(3) K_2 is correlated with u_1 (4) K_1 is correlated with u_1

87. Having known the last census population 'P_0' and growth rate 'r', the population after 'n' years based on compound interest formula will be :

(1) $\hat{P}_t = P_0(1+r)^n$ (2) $\hat{P}_t = P_0(1+n)^r$

(3) $\hat{P}_t = P_0/(1+r)^n$ (4) $\hat{P}_t = P_0/(1+n)^r$

88. Vital rates are customarily expressed as :

(1) Percentages (2) Per Thousand

(3) Per Million (4) Per Trillion

89. Age - Specific mortality rates fail to reveal :

(1) Mortality conditions (2) Age-distribution of persons

(3) Sex Ratio (4) All of the above

90. The relationship between NRR and GRR is :

(1) NRR and GRR are usually equal

(2) NRR can never exceed GRR

(3) NRR is generally greater than GRR

(4) None of the above

91. In SRSWOR, if $\bar{y} = 50$, $n = 100$, $N = 500$, then the estimated population total is :

(1) 250 (2) 500 (3) 25000 (4) 2500

92. Headquarters of Field Operations Division of NSSO are located at :

(1) New Delhi (2) Kolkata (3) Bombay (4) Chennai

93. The variance of stratified sampling mean $\overline{Y}_{st}$ is :

(1) $\sum_{h=1}^{L}\left(\frac{1}{N_h}-\frac{1}{n_h}\right)W_h^2 S_h^2$ (2) $\sum_{h=1}^{L}\left(\frac{1}{n_h}-\frac{1}{N_h}\right)W_h^2 S_h^2$

(3) $\sum_{h=1}^{L}\left(\frac{1}{N_h}-\frac{1}{n_h}\right)S_h^2$ (4) $\sum_{h=1}^{L}\left(\frac{1}{n_h}-\frac{1}{N_h}\right)W_h^2 S_h^2$

94. In simple random sampling, the biased of ratio estimator is unbiased if :

(1) They are independent (2) They are uncorrelated

(3) They are correlated (4) They are dependent

95. Wishart distribution (σ^2, n) follows :

(1) $\sigma^2\chi^2$ distribution
(2) $N(0, \sigma^2)$ distribution
(3) $\frac{e^{-n}\sigma^2}{n}$ distribution
(4) Beta distribution

96. Hotelling T^2 can be approximated to statistic 'F' with usual notation as test criteria following the inequality for rejection as :

(1) $T^2 > F_{p,n-p;\alpha}f$ or $n > p$
(2) $T^2 < F_{p,n-p;\alpha}f$ or $n > p$
(3) $T^2 > \frac{n-1}{n-p} F_{p,n-p;\alpha}f$ or $n > p$
(4) $T^2 > \frac{n-p}{n-1} F_{p,n-p;\alpha}f$ or $n > p$

97. A measure of association between a discriminant function and a set of dummy variables that define the group membership is known as :

(1) Multivariate Correlation
(2) Multicollinearity
(3) Canonical Correlation
(4) Biserial Correlation

98. The ratio of number of replications required in CRD and RBD for the same amount of information is :

(1) 3:2
(2) 5:3
(3) 5:4
(4) 3:5

99. In a Randomised Block Design, we always have :

(1) Number of blocks = Number of treatments
(2) Number of blocks > Number of treatments
(3) Number of blocks < Number of treatments
(4) All of the above

100. The additional effect gained due to combined effect of two or more factors is known as :

(1) Main Effect
(2) Interaction Effect
(3) Either (1) or (2)
(4) Neither (1) or (2)

Total No. of Printed Pages : 17

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Statistics

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Sr. No. ....

Time : **1¼ Hours** Max. Marks : **100** Total Questions : **100**

Roll No. (in figures)________________ (in words) __________________________

Name ________________________ Date of Birth__________________________

Father's Name ____________________ Mother's Name__________________________

Date of Examination______________________

________________________ ________________________

(Signature of the Candidate) (Signature of the Invigilator)

SEAL

CANDIDATES MUST READ THE FOLLOWING INFORMATION/INSTRUCTIONS BEFORE STARTING THE QUESTION PAPER.

1. ***All questions are compulsory.***
2. The candidates ***must return*** the question booklet as well as OMR Answer-Sheet to the Invigilator concerned before leaving the Examination Hall, failing which a case of use of unfair-means / mis-behaviour will be registered against him / her, in addition to lodging of an FIR with the police. Further the answer-sheet of such a candidate will not be evaluated.
3. Keeping in view the transparency of the examination system, carbonless OMR Sheet is provided to the candidate so that a copy of OMR Sheet may be kept by the candidate.
4. Question Booklet along with answer key of all the A, B, C & D code shall be got uploaded on the University Website immediately after the conduct of Entrance Examination. Candidates may raise valid objection/complaint if any, with regard to discrepancy in the question booklet/answer key within 24 hours of uploading the same on the University Website. The complaint be sent by the students to the Controller of Examinations by hand or through email. Thereafter, no complaint in any case, will be considered.
5. The candidate ***must not*** do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question booklet itself. Answers ***must not*** be ticked in the question booklet.
6. ***There will be no negative marking. Each correct answer will be awarded one full mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.***
7. Use only **Black** or **Blue Ball Point Pen** of good quality in the OMR Answer-Sheet.
8. *Before answering the questions, the candidates should ensure that they have been supplied correct and complete booklet. Complaints, if any, regarding misprinting etc. will not be entertained* ***30*** *minutes after starting of the examination.*

1. The pgf of a certain distribution is given as :

$P(s) = 3s^2 - 2s + 6$

What is the mean of this distribution ?

(1) 2 (2) 3 (3) 4 (4) 5

2. The relation between student's -t and F-distribution is :

(1) $F_{1,1} = t_n^2$ (2) $F_{n,1} = t_1^2$ (3) $F_{1,n} = t_\infty^2$ (4) $F_{1,n} = t_n^2$

3. The minimum variance unbiased estimator of θ^2 based on a sample of size 'n' from $N(\theta\ 1)$ is :

(1) $\overline{X}^2 - 1/n$ (2) $\overline{X}^2 + 1/n$

(3) $\sum(X_i - \overline{X}^2)/n$ (4) $\sum(X_i - \overline{X}^2)/(n-1)$

4. The Bayes estimator of a parameter under squared error loss function is :

(1) Posterior mean (2) Posterior median

(3) Posterior mode (4) Posterior variance

5. The decision criteria in SPRT depends on the function of :

(1) Type I error

(2) Type II error

(3) Both type I and type II error

(4) Neither Type I nor Type II error

6. What is an unbiased estimator of θ for the distribution $f(x, \theta) = \theta e^{-\theta} x, x \geq 0$?

(1) $\frac{(n-1)\overline{X}}{n}$ (2) $\frac{(n-1)}{n\overline{X}}$ (3) $\frac{\overline{X}}{n-1}$ (4) $\frac{1}{(n-1)\overline{X}}$

7. Given a random sample :

$f(x, \theta) = \frac{2}{\alpha^2}(\alpha - x), 0 < x < \alpha$

What is the MLE of α ?

(1) X (2) $2X$ (3) $\frac{X^2}{2}$ (4) $\frac{X}{2}$

8. For a particular hypothesis test, the probabilities of type I and type II errors are respectively, 0.05 and 0.09. The power of the test is :

(1) 0.95 (2) 0.14 (3) 0.86 (4) 0.91

9. A random sample of 100 articles are taken from a batch of 2000 articles shows that the average diameter of the articles is 0.354 and a standard deviation 0.048. What is the 95% confidence interval for the average diameter of the batch ?

(1) (0.2934,0.4235) (2) (0.3448,0.3632)
(3) (0.3021,0.3824) (4) (0.3923,0.4212)

10. Which of the following is ***true*** ?

(1) Unbiased estimator is always efficient.

(2) Consistent estimator is always unbiased.

(3) Unbiased estimator is always consistent.

(4) MLE is always a function of sufficient statistic.

11. Service time in queueing theory is usually assumed to follow :

(1) Poisson Distribution (2) Erlang Distribution
(3) Exponential Distribution (4) Normal Distribution

12. For M|M|1 queueing system, the expected number of customers in systems are :

(1) $L_S = \left(\frac{\lambda}{\mu - \lambda}\right)$ (2) $L_S = \left(\frac{\lambda - \mu}{\lambda}\right)$

(3) $L_S = \left(\frac{\mu}{\mu - \lambda}\right)$ (4) $L_S = \left(\frac{\mu - \lambda}{\lambda}\right)$

13. Which of the following relationships is ***not*** true ?

(1) $W_S = W_q + \frac{1}{\mu}$ (2) $L_S = \lambda W_S$

(3) $L_S = L_q + \frac{1}{\lambda}$ (4) $L_q = \lambda W_q$

14. Maximize $Z = 10x_1 + 25x_2$, subject to $0 \le x_1 \le 3, 0 \le x_2 \le 3, x_1 + x_2 \le 5$

(1) 80 at (3, 2) (2) 75 at (0,3) (3) 30 at (3,0) (4) 95 at (2, 3)

15. Which variable is added for the less than or equal to type of constraint ?

(1) Slack (2) Surplus (3) Artificial (4) Basic

16. The convex combination of two points $\bar{x}_1,\ \bar{x}_2 \in X$ is referred as :

(1) $(1-\lambda)\bar{x}_1 + \lambda\bar{x}_2,\ 0 \le \lambda \le 1$

(2) $(1-\lambda)\bar{x}_1 + \lambda\bar{x}_2$, λ is any real number

(3) $\bar{x}_1 + \lambda\bar{x}_2,\ 0 \le \lambda \le 1$

(4) $\lambda\bar{x}_1 + \lambda\bar{x}_2$, λ is any real number

17. The assignment problem is :

(1) non-linear programming problem

(2) dynamic programming problem

(3) integer linear programming problem

(4) integer non-linear programming problem

18. The order of Convergence of Secant Method is :

(1) 2.4 (2) 2 (3) 1.62 (4) 1

19. If $f(0) = 3, f(1) = 5, f(3) = 21$, then the unique polynomials of degree 2 or less using Newton divided difference interpolation will be :

(1) $2x^2 + 2x + 1$ (2) $2x^2 - 3x + 1$

(3) $2x^2 + 3$ (4) $x^2 + 3x + 2$

20. The third difference of a cubic $\Delta^3 y$ function are :

(1) Constant (2) Not constant

(3) Variables (4) None of the above

21. Let Y_1 , Y_2 , Y_3 be uncorrelated observations with common variance σ^2 and expectations given by $(Y_1) = \beta_1$, $E(Y_2) = \beta_2$ and $E(Y_3) = \beta_1 + \beta_2$ where β_1 and β_2 are unknown parameters. The best linear unbiased estimator of $\beta_1 + \beta_2$ is :

(1) $\frac{1}{2}(Y_1+Y_2+Y_3)$ (2) $Y_1 + Y_2$

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22. Under the regulatory conditions, if λ_n is the likelihood ratio, then the asymptotic distribution of $-2\log\lambda_n$ as $n \to \infty$ is :

(1) Normal distribution (2) Chi square distribution

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23. If the pdf of a random variable 'X' is $f(x, \theta) = 1 - \theta^2, 0 \le x \le \frac{1}{1-\theta^2}$ and if we reject $H_0 : \theta = \frac{1}{2}$ against the alternative $H_1 : \theta = \frac{3}{4}$ whenever $x \ge 1$, then what is the power of the test ?

(1) $\frac{15}{16}$ (2) $\frac{1}{2}$ (3) $\frac{7}{16}$ (4) $\frac{9}{16}$

24. If $X_1, X_2, \ldots\ldots X_n$ is a random sample form poisson distribution with parameter 'λ', then the maximum likelihood estimator of $\log\lambda$ is given by :

(1) $e^{\left(\sum_{i=1}^{n}\frac{X_i}{n}\right)}$ (2) $\log\left(\sum_{i=1}^{n}\frac{X_i}{n}\right)$ (3) $e^{\left(\sum_{i=1}^{n}X_i\right)}$ (4) $\log\left(\sum_{i=1}^{n}X_i\right)$

25. Which of the following could be used as a test for autocorrelation up to third order ?

(1) Spearmen Correlation test (2) The Breusch-Goldfrey test

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$$K_1 = \alpha_1 K_2 + \beta_1 Z_1 + u_1$$

$$K_2 = \alpha_2 K_1 + \beta_2 Z_2 + u_2$$

OLS will suffer from simultaneous bias if :

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(3) K_2 is correlated with u_1 (4) K_1 is correlated with u_1

27. Having known the last census population 'P_0' and growth rate 'r', the population after 'n' years based on compound interest formula will be :

(1) $\hat{P}_t = P_0(1+r)^n$ (2) $\hat{P}_t = P_0(1+n)^r$

(3) $\hat{P}_t = P_0/(1+r)^n$ (4) $\hat{P}_t = P_0/(1+n)^r$

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(1) Mortality conditions (2) Age-distribution of persons

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30. The relationship between NRR and GRR is :

(1) NRR and GRR are usually equal

(2) NRR can never exceed GRR

(3) NRR is generally greater than GRR

(4) None of the above

31. If the random variables X, Y and Z have the means $\mu_x = 5$, $\mu_y = 7$ and $\mu_z = 4$; $\sigma_X^2 = 10$, $\sigma_Y^2 = 14$ and $\sigma_Z^2 = 20$; Cov(X, Y) = 1, Cov (X, Z) = 3 and Cov(Y, Z) = 2, then what is the covariance of $U = X + 4Y + 2Z$ and $V = 3X - Y - Z$?

(1) −76 (2) 82 (3) −82 (4) 76

32. Neelam has appeared in an examination which follows multiple choice questions, each having five possible answers. The probability that she knows an answer is 0.75. If she does not know an answer, she will guess, with the conditional probability 1/5 of being correct. The conditional probability that Neelam knows the answer, given that she gives the correct answer is :

(1) 0.25 (2) 0.80 (3) 0.90 (4) 0.94

33. A drunkard takes a forward step with probability 'p' and a backward step with probability 'q'. After taking 11 steps, the probability that he is the one step away from the starting point is :

(1) $p^6 + q^6$ (2) $2(p^6 + q^5)$

(3) $462\, p^6 q^5$ (4) $462\, p^5 q^5$

34. For the sequence $\{X_n\}$ of independent random variables the following are defined :

$P(X_k = \pm 2^k) = 2^{-(2k+1)}; P(X_k = 0) = 1 - 2^{-2k}$

Which of the following are not correct(s) ?

(a) WLLN holds because $E(X_k) = 0$ and $\lim_{n\to\infty} \frac{B_n}{n^2} \to 0$ where $B_n = \text{Var}\left(\Sigma_{i=1}^{n} X_i\right)$

(b) Weak law of large numbers holds by Khinchin's theorem because $E(X_k)$ is finite.

(c) WLLN holds since X_k are identically distributed.

Select the correct answer using code given below :

(1) (b) and (c) only (2) (a) and (b) only

(3) (a) and (c) only (4) (a), (b) and (c) only

35. If σ_X^2, σ_Y^2 and σ_{X-Y}^2 are the variances of X, Y and $X-Y$ respectively, then what is the coefficient of correlation between 'X' and 'Y' ?

(1) $\frac{\sigma_X^2 + \sigma_Y^2 - \sigma_{X-Y}^2}{2\sigma_X\sigma_Y}$ (2) $\frac{\sigma_X^2 + \sigma_Y^2}{2\sigma_X\sigma_Y}$

(3) $\frac{\sigma_X^2 + \sigma_Y^2 - \sigma_{X+Y}^2}{2\sigma_X\sigma_Y}$ (4) $\frac{\sigma_X^2 - \sigma_Y^2}{2\sigma_X\sigma_Y}$

36. Let 'X' and 'Y' be independent Gamma $G(\alpha_1, \beta)$ and $G(\alpha_2, \beta)$ random variables respectively. Then $\frac{X}{X+Y}$ is distributed as :

(1) $G(\alpha_1 + \alpha_2, \beta)$ (2) $\beta_1(\alpha_1, \alpha_2)$

(3) $U(0, 1)$ (4) $G(\alpha_1, \alpha_2)$

37. The Joint Density Function of 'X' and 'Y' is given by :

$$f(x,y) = \begin{cases} 2e^{-x}e^{-2y} & 0 < x < \infty, 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

What is the value of P(X < Y) ?

(1) 1/3 (2) 1/2 (3) 1/4 (4) 1/6

38. For a certain frequency distribution, the numerical computation yields the following :

Mean = 62, Median = 65 , Coefficient of skewness = –0.3, then the standard deviation is equal to :

(1) 10 (2) 30 (3) 90 (4) 300

39. Let 'X' be a random variable having Probability Density Function :

$$f(x) = \begin{cases} x/2 & 0 < x < 1 \\ 1/2 & 1 < x \le 2 \\ (3-x)/3 & 2 < x \le 3 \end{cases}$$

Then P(1.5 < X < 2.5|X > 1) equals to :

(1) 3/8 (2) 5/8 (3) 1/2 (4) 1/4

40. Let X_1, X_2, ... X_n be i.i.d. random variables with $E(X_i) = \mu$ and $E(X_i^2) < \infty$, then the consistent estimator for μ is :

(1) $\dfrac{2i^2}{n(n+1)\sum X_i}$ (2) $\dfrac{2}{n(n+1)}\sum iX_i$

(3) $\dfrac{2i}{n(n+1)}\sum X_i$ (4) $\dfrac{2}{n(n+1)}\sum i^2 X_i$

41. The correct relationship between A.M, G.M., and H.M. is :

(1) A.M.= G.M.= H.M. (2) G.M. ≥ A.M. ≥ H.M.

(3) H.M. ≥ G.M. ≥ A.M. (4) A.M. ≥ G.M. ≥ H.M

42. Average wages of workers of factory are Rs. 550 per month and the standard deviation of wages is 110. The coefficients of variation is :

(1) 30% (2) 15% (3) 500% (4) 20%

43. If the mode of a frequency distribution is 16 and its mean is 16, then the median of the distribution is :

(1) 0 (2) 16 (3) 32 (4) 8

44. If Quartile deviation of a set of observations is given as 6.4 and the value of first quartile is 5. What is the value of the third quartile ?

(1) 12 (2) 15.3 (3) 17.8 (4) 20.2

45. For a leptokurtic frequency curve, the measures of kurtosis is :

(1) 0 (2) −3 (3) less than 1 (4) greater than 3

46. Standard error of the sample correlation coefficient 'r' is based on 'n' paired values is :

(1) $\frac{1+r^2}{\sqrt{n}}$ (2) $\frac{1+r^2}{n}$

(3) $\frac{1-r^2}{\sqrt{n}}$ (4) $\frac{1+r^2}{\sqrt{n-1}}$

47. Given the following set of equations :

$$x_1 + 4x_2 - x_3 = 3$$
$$5x_1 + 2x_2 + 3x_3 = 4$$

The basic feasible solution involving x_1 and x_2 is :

(1) $\left(\frac{5}{9}, \frac{11}{18}, 0\right)$ (2) $\left(\frac{5}{9}, 0, 0\right)$

(3) $\left(0, \frac{11}{18}, 0\right)$ (4) $\left(\frac{2}{9}, 0, \frac{3}{14}\right)$

48. The range of a partial correlation coefficient is :

(1) 0 to ∞ (2) −∞ to ∞ (3) 0 to 1 (4) −1 to 1

49. The term regression was introduced by :

(1) Sir Francis Galton (2) R.A. Fisher

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$$6y = 5x + 90$$
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(1) New Delhi (2) Kolkata (3) Bombay (4) Chennai

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(1) $\sum_{h=1}^{L}\left(\frac{1}{N_h} - \frac{1}{n_h}\right)W_h^2 S_h^2$

(2) $\sum_{h=1}^{L}\left(\frac{1}{n_h} - \frac{1}{N_h}\right)W_h^2 S_h^2$

(3) $\sum_{h=1}^{L}\left(\frac{1}{N_h} - \frac{1}{n_h}\right)S_h^2$

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(3) $\frac{e^{-n}\sigma^2}{n}$ distribution (4) Beta distribution

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(1) $T^2 > F_{p,n-p;\alpha}f$ or $n > p$

(2) $T^2 < F_{p,n-p;\alpha}f$ or $n > p$

(3) $T^2 > \frac{n-1}{n-p}F_{p,n-p;\alpha}f$ or $n > p$

(4) $T^2 > \frac{n-p}{n-1}F_{p,n-p;\alpha}f$ or $n > p$

57. A measure of association between a discriminant function and a set of dummy variables that define the group membership is known as :

(1) Multivariate Correlation
(2) Multicollinearity
(3) Canonical Correlation
(4) Biserial Correlation

58. The ratio of number of replications required in CRD and RBD for the same amount of information is :

(1) 3:2
(2) 5:3
(3) 5:4
(4) 3:5

59. In a Randomised Block Design, we always have :

(1) Number of blocks = Number of treatments
(2) Number of blocks > Number of treatments
(3) Number of blocks < Number of treatments
(4) All of the above

60. The additional effect gained due to combined effect of two or more factors is known as :

(1) Main Effect
(2) Interaction Effect
(3) Either (1) or (2)
(4) Neither (1) or (2)

61. The mean of Binomial distribution $B(n, p)$ is :

(1) np
(2) $np - 1$
(3) $np - 2$
(4) $np - 3$

62. Given that $E[X + 4] = 10$ and $E[X + 4]^2 = 116$, then Var$[X]$ is equal to :

(1) 4
(2) 8
(3) 12
(4) 16

63. Let 'X' be a continuous random variable with Probability Density Function

$$f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ -ax + 3a, & 2 \le x \le 3 \\ 0, & x > 3 \end{cases}$$

Then the value of 'a' is given by :

(1) 0.4
(2) 0.5
(3) 0.3
(4) 0.1

64. Let 'X' be a continuous random variable with Probability Density Function

$$f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ -ax + 3a, & 2 \le x \le 3 \\ 0, & x > 3 \end{cases}$$

Then the value of $P(X \le 1.5)$ is given by :

(1) 0.5 (2) 0.81 (3) 0.19 (4) 0.17

65. The moment generating function of Poisson Distribution is :

(1) $e^{\lambda}(e^{t}-1)$ (2) $e^{t}(e^{\lambda}-1)$ (3) $e^{t}-1$ (4) $e^{\lambda}(1-e^{t})$

66. Which of the following is the median of the exponential distribution with parameter λ ?

(1) λ (2) $-\lambda^{-1}$ (3) λ^{-1} (4) λ^{-2}

67. The Quartile Deviation of the normal distribution is :

(1) Q. D. = $\frac{2}{4}\sigma$ (2) Q. D. = $\frac{2}{3}\sigma$ (3) Q. D. = $\frac{2}{5}\sigma$ (4) Q. D. = $\frac{3}{4}\sigma$

68. If A and B are two independent events, then $P(\overline{A} \cap \overline{B})$ is equal to :

(1) $P(\overline{A})P(\overline{B})$ (2) $1 - P(A \cup B)$

(3) $[1 - P(A)][1 - P(B)]$ (4) All of the above

69. If event A and event B has occurred and it is known that $P(B) = 1$, the conditional probability $P(A/B)$ is equal to :

(1) $P(A)$ (2) $P(B)$ (3) One (4) Zero

70. The recurrence formula for geometric distribution is given by :

(1) $p(x + i) = qp(x + i)$ (2) $p(x) = qp(x + i)$

(3) $p(x + i) = qp(x)$ (4) $p(x) = p(x + i)$

71. For a standard $n \times n$ Latin Square, how many different Latin squares can be obtained with the same standard ?

(1) $n!\,(n-2)!$ (2) $(n-1)!\,(n-2)!$

(3) $n!\,(n-1)!$ (4) $n!\,(n+1)!$

72. The condition for the time reversal test to hold good with usual notations is :

(1) $P_{01} \times P_{10} = 1$

(2) $P_{10} \times P_{01} = 0$

(3) $P_{01} / P_{10} = 1$

(4) $P_{01} + P_{10} = 1$

73. Weight in Laspeyre's price index number is known as ....

(1) Quantity during the current year

(2) Quantity in the base year

(3) Price during the current year

(4) Price in the base year

74. In India, the collection of vital statistics started for first time in :

(1) 1720 (2) 1886 (3) 1969 (4) 1946

75. Fisher's ideal formula does not satisfy ....

(1) Time Reversal Test

(2) Circular Test

(3) Factor Reversal Test

(4) Unit Test

76. If l_x is the number of persons living at the age 'x' and 'L_x' the number of persons living in the mid of 'x' and '$(x+1)$' years, then the relation between l_x and L_x is :

(1) $L_x = \frac{1}{2}(l_x + l_{x+1})$

(2) $L_x = \left(\frac{x}{2} + l_x\right)$

(3) $L_x = l_{x+\frac{1}{2}}$

(4) $L_x = l_{x+\frac{3}{2}}$

77. Fertility rates mainly depend on :

(1) Total female population

(2) Total population

(3) Female population of child bearing age

(4) Number of newly born babies

78. If the quantity demanded of a commodity is unresponsive to change in prices, then the demand of that commodity is :

(1) Perfectly Inelastic

(2) Elastic

(3) Unit Elastic

(4) Inelastic

79. The elasticity for the demand of the durable goods is :

(1) Zero (2) Equal to unity

(3) Greater than unity (4) Less than Unity

80. Consider the following matrix :

$$P = \begin{bmatrix} 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \\ 1/2 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \end{bmatrix}$$

If P is a stochastic matrix, then which of the following is ***not*** true ?

(1) P is Ergodic (2) P is Regular

(3) P is not Regular (4) Both (1) and (3)

81. Construction of life tables is based on the assumption that :

(1) Age - specific death rates are constant at all ages.

(2) Death rates are uniformly distributed between two birth days.

(3) Mortality rates are same for male and female populations.

(4) All of the above

82. A life table consists of :

(1) Seven columns (2) Eight columns

(3) Six columns (4) Nine Columns

83. King's abridged life tables are based in the calculation of :

(1) Central mortality rate

(2) The number of persons and deaths for central age in the interval $\{x, x + n\}$

(3) Both (1) and (2)

(4) Neither (1) nor (2)

84. If Laspeyre's price index number is 324 and Paasche's Price Index number is 144, then Fisher's Ideal Index Number is :

(1) 180 (2) 234 (3) 216 (4) 222

85. If a negative value appears in the solution values (X_B) column of the simplex table, then :

(1) The solution is optimal

(2) The solution is infeasible

(3) The solution is unbounded

(4) All of the above

86. The maximum number of extreme points for a LPP

Max Z = cx

Subject to

$Ax = b; x \geq 0$

Where A is $m \times n$ matrix is equal to :

(1) $\frac{m!}{n!(m-n)!}$ (2) $\frac{n!}{m!(n-m)!}$

(3) $(m-n)$ (4) mn

87. In a basic feasible solution of an $m \times n$ transportation problem, the number of positive allocations is atmost :

(1) $m+n$ (2) $m+n-1$

(3) $m-n$ (4) $m+n-2$

88. The necessary and sufficient condition for the existence of a feasible solution of a transportation problem is :

(1) $\sum a_i = \sum b_j$ (2) $\sum a_i \neq \sum b_j$

(3) $\sum a_i = 0$ (4) $\sum b_j = 0$

89. Consider the linear programming problem :

Max $Z = 3x_1 + 2x_2$

Subject to

$x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0$

Then its solution is :

(1) $x_1 = 3,\ x_2 = 1$, Max $Z = 11$

(2) $x_1 = 1,\ x_2 = 3$, Max $Z = 10$

(3) $x_1 = 2,\ x_2 = 1$, Max $Z = 11$

(4) $x_1 = 1,\ x_2 = 2$, Max $Z = 10$

90. The probability of living of a person in the age group 'x' to '$(x + n)$' can be obtained by the formula :

(1) l_{x+n} / l_x

(2) $(l_x - l_{x+n}) / l_{x+n}$

(3) $(l_x - l_{x+n}) / l_x$

(4) l_x / l_{x+n}

91. Let 'f' be a non-constant entire function. Which of the following properties is possible for 'f' for each $z \in \mathbb{C}$?

(1) Re f(z) = Im f(z)

(2) $|f(z)| < 1$

(3) Im(z) < 0

(4) $f(z) \neq 0$

92. Let 'f' and 'g' be mesomorphic function on $\mathbb{C}$. If 'f' has a zero of order 'k' at $z = a$ and z has a pole of order 'm' at z = 0, then g(f(z)) has :

(1) a zero of order km at $z = a$

(2) a pole of order km at $z = a$

(3) a zero of order $|k - m|$ at $z = a$

(4) a pole of order $|k - m|$ at $z = a$

93. The set of vectors $X_1 = (2, 1, 4)$, $X_2 = (-3, 2, -1)$, $X_3 = (1, -3, -2)$ is :

(1) Linearly dependent

(2) Linearly independent

(3) Both of them

(4) None of these

94. Let a be 2 × 2 with Det(A) = 1 & Trace(A) = 3, then Trace(A^2) is :

(1) 7 (2) 8 (3) 9 (4) 10

95. If A is (2 × 2) matrix over IR with Det(A + I) = 1 + Det (A), then we can conclude that :

(1) Det(A) = 0
(2) A = 0
(3) Tr(A) = 0
(4) A is non-singular

96. If A is a square matrix, then $A + A'$, AA' and $A'A$ are :

(1) Symmetric
(2) Skew Symmetric
(3) Hermitian
(4) Skew Hermitian

97. If $\lim\limits_{n\to\infty} a_n^{1/n} = \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n}$ provided limit on RHS exists. This result is known as :

(1) Cauchy 1st Theorem on Limits
(2) Cauchy 2nd Theorem on Limits
(3) Squeeze Principle
(4) Leibnitz' Rule

98. $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ is :

(1) 1 (2) 0 (3) e (4) 2

99. Consider the sequence $a_n = \left(1+(-1)^n \frac{1}{n}\right)^n$ then :

(1) $\lim\limits_{n\to\infty} \text{Sup}(a_n) = \lim\limits_{n\to\infty} \text{Inf}(a_n) = 1$

(2) $\lim\limits_{n\to\infty} \text{Sup}(a_n) = \lim\limits_{n\to\infty} \text{Inf}(a_n) = e$

(3) $\lim\limits_{n\to\infty} \text{Sup}(a_n) = \lim\limits_{n\to\infty} \text{Inf}(a_n) = \frac{1}{e}$

(4) $\lim\limits_{n\to\infty} \text{Sup}(a_n) = e, \lim\limits_{n\to\infty} \text{Inf}(a_n) = \frac{1}{e}$

100. If X and Y are standard normal variates with correlation coefficient 'ρ' between them, then the correlation coefficient between X^2 and Y^2 is :

(1) $2\rho - 1$ (2) ρ^2 (3) ρ (4) $\sqrt{\rho}$

Total No. of Printed Pages : 17

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Sr. No. ....

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Name ______________________ Date of Birth______________________

Father's Name __________________ Mother's Name____________________

Date of Examination________________________

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(Signature of the Candidate) (Signature of the Invigilator)

SEAL

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3. Keeping in view the transparency of the examination system, carbonless OMR Sheet is provided to the candidate so that a copy of OMR Sheet may be kept by the candidate.
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5. The candidate ***must not*** do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question booklet itself. Answers ***must not*** be ticked in the question booklet.
6. ***There will be no negative marking. Each correct answer will be awarded one full mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.***
7. Use only **Black** or **Blue Ball Point Pen** of good quality in the OMR Answer-Sheet.
8. *Before answering the questions, the candidates should ensure that they have been supplied correct and complete booklet. Complaints, if any, regarding misprinting etc. will not be entertained* **30** *minutes after starting of the examination.*

1. Construction of life tables is based on the assumption that :

(1) Age - specific death rates are constant at all ages.

(2) Death rates are uniformly distributed between two birth days.

(3) Mortality rates are same for male and female populations.

(4) All of the above

2. A life table consists of :

(1) Seven columns (2) Eight columns

(3) Six columns (4) Nine Columns

3. King's abridged life tables are based in the calculation of :

(1) Central mortality rate

(2) The number of persons and deaths for central age in the interval $\{x, x + n\}$

(3) Both (1) and (2)

(4) Neither (1) nor (2)

4. If Laspeyre's price index number is 324 and Paasche's Price Index number is 144, then Fisher's Ideal Index Number is :

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5. If a negative value appears in the solution values (X_B) column of the simplex table, then :

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(2) The solution is infeasible

(3) The solution is unbounded

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6. The maximum number of extreme points for a LPP

Max $Z = cx$

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Where A is $m \times n$ matrix is equal to :

(1) $\frac{m!}{n!(m-n)!}$ (2) $\frac{n!}{m!(n-m)!}$ (3) $(m-n)$ (4) mn

7. In a basic feasible solution of an $m \times n$ transportation problem, the number of positive allocations is atmost :

(1) $m+n$ (2) $m+n-1$ (3) $m-n$ (4) $m+n-2$

8. The necessary and sufficient condition for the existence of a feasible solution of a transportation problem is :

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9. Consider the linear programming problem :

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Subject to

$x_1 + x_2 \leq 4$

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Then its solution is :

(1) $x_1 = 3,\ x_2 = 1$, Max $Z = 11$ (2) $x_1 = 1,\ x_2 = 3$, Max $Z = 10$

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10. The probability of living of a person in the age group 'x' to '$(x + n)$' can be obtained by the formula :

(1) l_{x+n} / l_x (2) $(l_x - l_{x+n}) / l_{x+n}$

(3) $(l_x - l_{x+n}) / l_x$ (4) l_x / l_{x+n}

11. In SRSWOR, if $\bar{y} = 50, n = 100, N = 500$, then the estimated population total is :

(1) 250 (2) 500 (3) 25000 (4) 2500

12. Headquarters of Field Operations Division of NSSO are located at :

(1) New Delhi (2) Kolkata (3) Bombay (4) Chennai

13. The variance of stratified sampling mean $\bar{Y}_{st}$ is :

(1) $\sum_{h=1}^{L}\left(\frac{1}{N_h} - \frac{1}{n_h}\right)W_h^2 S_h^2$ (2) $\sum_{h=1}^{L}\left(\frac{1}{n_h} - \frac{1}{N_h}\right)W_h^2 S_h^2$

(3) $\sum_{h=1}^{L}\left(\frac{1}{N_h} - \frac{1}{n_h}\right)S_h^2$ (4) $\sum_{h=1}^{L}\left(\frac{1}{n_h} - \frac{1}{N_h}\right)W_h S_h^2$

14. In simple random sampling, the biased of ratio estimator is unbiased if :

(1) They are independent (2) They are uncorrelated

(3) They are correlated (4) They are dependent

15. Wishart distribution (σ^2, n) follows :

(1) $\sigma^2\chi^2$ distribution (2) $N(0, \sigma^2)$ distribution

(3) $\frac{e^{-n}\sigma^2}{n}$ distribution (4) Beta distribution

16. Hotelling T^2 can be approximated to statistic 'F' with usual notation as test criteria following the inequality for rejection as :

(1) $T^2 > F_{p,n-p;\alpha} f$ or $n > p$ (2) $T^2 < F_{p,n-p;\alpha} f$ or $n > p$

(3) $T^2 > \frac{n-1}{n-p} F_{p,n-p;\alpha} f$ or $n > p$ (4) $T^2 > \frac{n-p}{n-1} F_{p,n-p;\alpha} f$ or $n > p$

17. A measure of association between a discriminant function and a set of dummy variables that define the group membership is known as :

(1) Multivariate Correlation (2) Multicollinearity

(3) Canonical Correlation (4) Biserial Correlation

18. The ratio of number of replications required in CRD and RBD for the same amount of information is :

(1) 3:2 (2) 5:3 (3) 5:4 (4) 3:5

19. In a Randomised Block Design, we always have :

(1) Number of blocks = Number of treatments

(2) Number of blocks > Number of treatments

(3) Number of blocks < Number of treatments

(4) All of the above

20. The additional effect gained due to combined effect of two or more factors is known as :

(1) Main Effect (2) Interaction Effect

(3) Either (1) or (2) (4) Neither (1) or (2)

21. If the random variables X, Y and Z have the means $\mu_x = 5$, $\mu_y = 7$ and $\mu_z = 4$; $\sigma_X^2 = 10$, $\sigma_Y^2 = 14$ and $\sigma_Z^2 = 20$; Cov(X, Y) = 1, Cov (X, Z) = 3 and Cov(Y, Z) = 2, then what is the covariance of $U = X + 4Y + 2Z$ and $V = 3X - Y - Z$?

(1) −76 (2) 82 (3) −82 (4) 76

22. Neelam has appeared in an examination which follows multiple choice questions, each having five possible answers. The probability that she knows an answer is 0.75. If she does not know an answer, she will guess, with the conditional probability 1/5 of being correct. The conditional probability that Neelam knows the answer, given that she gives the correct answer is :

(1) 0.25 (2) 0.80 (3) 0.90 (4) 0.94

23. A drunkard takes a forward step with probability 'p' and a backward step with probability 'q'. After taking 11 steps, the probability that he is the one step away from the starting point is :

(1) $p^6 + q^6$ (2) $2(p^6 + q^5)$

(3) $462\, p^6 q^5$ (4) $462\, p^5 q^5$

24. For the sequence $\{X_n\}$ of independent random variables the following are defined :

$P(X_k = \pm 2^k) = 2^{-(2k+1)}$; $P(X_k = 0) = 1 - 2^{-2k}$

Which of the following are not correct(s) ?

(a) WLLN holds because $E(X_k) = 0$ and $\lim\limits_{n \to \infty} \frac{B_n}{n^2} \to 0$ where $B_n = \text{Var}\left(\Sigma_{i=1}^{n} X_i\right)$

(b) Weak law of large numbers holds by Khinchin's theorem because $E(X_k)$ is finite.

(c) WLLN holds since X_k are identically distributed.

Select the correct answer using code given below :

(1) (b) and (c) only (2) (a) and (b) only

(3) (a) and (c) only (4) (a), (b) and (c) only

25. If σ_X^2, σ_Y^2 and σ_{X-Y}^2 are the variances of X, Y and $X - Y$ respectively, then what is the coefficient of correlation between 'X' and 'Y' ?

(1) $\dfrac{\sigma_X^2 + \sigma_Y^2 - \sigma_{X-Y}^2}{2\sigma_X \sigma_Y}$ (2) $\dfrac{\sigma_X^2 + \sigma_Y^2}{2\sigma_X \sigma_Y}$

(3) $\dfrac{\sigma_X^2 + \sigma_Y^2 - \sigma_{X+Y}^2}{2\sigma_X \sigma_Y}$ (4) $\dfrac{\sigma_X^2 - \sigma_Y^2}{2\sigma_X \sigma_Y}$

26. Let 'X' and 'Y' be independent Gamma $G(\alpha_1, \beta)$ and $G(\alpha_2, \beta)$ random variables respectively. Then $\dfrac{X}{X+Y}$ is distributed as :

(1) $G(\alpha_1 + \alpha_2, \beta)$ (2) $\beta_1(\alpha_1, \alpha_2)$

(3) $U(0, 1)$ (4) $G(\alpha_1, \alpha_2)$

27. The Joint Density Function of 'X' and 'Y' is given by :

$$f(x, y) = \begin{cases} 2e^{-x}e^{-2y} & 0 < x < \infty, 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

What is the value of P(X < Y) ?

(1) 1/3 (2) 1/2 (3) 1/4 (4) 1/6

28. For a certain frequency distribution, the numerical computation yields the following :

Mean = 62, Median = 65 , Coefficient of skewness = –0.3, then the standard deviation is equal to :

(1) 10 (2) 30 (3) 90 (4) 300

29. Let 'X' be a random variable having Probability Density Function :

$$f(x) = \begin{cases} x/2 & 0 < x < 1 \\ 1/2 & 1 < x \le 2 \\ (3-x)/3 & 2 < x \le 3 \end{cases}$$

Then P(1.5 < X < 2.5|X > 1) equals to :

(1) 3/8 (2) 5/8 (3) 1/2 (4) 1/4

30. Let $X_1, X_2, \ldots\ldots X_n$ be i.i.d. random variables with $E(X_i) = \mu$ and $E(X_i^2) < \infty$, then the consistent estimator for μ is :

(1) $\frac{2i^2}{n(n+1)\sum X_i}$ (2) $\frac{2}{n(n+1)}\sum iX_i$

(3) $\frac{2i}{n(n+1)}\sum X_i$ (4) $\frac{2}{n(n+1)}\sum i^2 X_i$

31. The pgf of a certain distribution is given as :

$P(s) = 3s^2 - 2s + 6$

What is the mean of this distribution ?

(1) 2 (2) 3 (3) 4 (4) 5

32. The relation between student's -t and F-distribution is :

(1) $F_{1,1} = t_n^2$ (2) $F_{n,1} = t_1^2$

(3) $F_{1,n} = t_\infty^2$ (4) $F_{1,n} = t_n^2$

33. The minimum variance unbiased estimator of θ^2 based on a sample of size 'n' from $N(\theta\ 1)$ is :

(1) $\bar{X}^2 - 1/n$ (2) $\bar{X}^2 + 1/n$

(3) $\sum(X_i - \bar{X}^2)/n$ (4) $\sum(X_i - \bar{X}^2)/(n-1)$

34. The Bayes estimator of a parameter under squared error loss function is :

(1) Posterior mean (2) Posterior median

(3) Posterior mode (4) Posterior variance

35. The decision criteria in SPRT depends on the function of :

(1) Type I error (2) Type II error

(3) Both type I and type II error (4) Neither Type I nor Type II error

36. What is an unbiased estimator of θ for the distribution $f(x, \theta) = \theta e^{-\theta} x, x \geq 0$?

(1) $\frac{(n-1)\overline{X}}{n}$ (2) $\frac{(n-1)}{n\overline{X}}$ (3) $\frac{\overline{X}}{n-1}$ (4) $\frac{1}{(n-1)\overline{X}}$

37. Given a random sample :

$$f(x,\theta) = \frac{2}{\alpha^2}(\alpha - x), 0 < x < \alpha$$

What is the MLE of α ?

(1) X (2) $2X$ (3) $\frac{X^2}{2}$ (4) $\frac{X}{2}$

38. For a particular hypothesis test, the probabilities of type I and type II errors are respectively, 0.05 and 0.09. The power of the test is :

(1) 0.95 (2) 0.14 (3) 0.86 (4) 0.91

39. A random sample of 100 articles are taken from a batch of 2000 articles shows that the average diameter of the articles is 0.354 and a standard deviation 0.048. What is the 95% confidence interval for the average diameter of the batch ?

(1) (0.2934,0.4235) (2) (0.3448,0.3632)

(3) (0.3021,0.3824) (4) (0.3923,0.4212)

40. Which of the following is ***true*** ?

(1) Unbiased estimator is always efficient.

(2) Consistent estimator is always unbiased.

(3) Unbiased estimator is always consistent.

(4) MLE is always a function of sufficient statistic.

41. Let 'f' be a non-constant entire function. Which of the following properties is possible for 'f' for each $z \in \mathbb{C}$?

(1) Re f(z) = Im f(z) (2) |f(z)| < 1

(3) Im(z) < 0 (4) f(z) ≠ 0

42. Let 'f' and 'g' be mesomorphic function on ($\mathbb{C}$. If 'f' has a zero of order 'k' at $z = a$ and z has a pole of order 'm' at z = 0, then g(f(z)) has :

(1) a zero of order km at $z = a$

(2) a pole of order km at $z = a$

(3) a zero of order |k − m| at $z = a$

(4) a pole of order |k− m| at $z = a$

43. The set of vectors $X_1 = (2, 1, 4)$, $X_2 = (-3, 2, -1)$, $X_3 = (1, -3, -2)$ is :

(1) Linearly dependent (2) Linearly independent

(3) Both of them (4) None of these

44. Let a be 2 × 2 with Det(A) = 1 & Trace(A) = 3, then Trace(A^2) is :

(1) 7 (2) 8 (3) 9 (4) 10

45. If A is (2 × 2) matrix over IR with Det(A + I) = 1 + Det (A), then we can conclude that :

(1) Det(A) = 0 (2) A = 0

(3) Tr(A) = 0 (4) A is non-singular

46. If A is a square matrix, then $A + A'$, AA' and $A'A$ are :

(1) Symmetric (2) Skew Symmetric

(3) Hermitian (4) Skew Hermitian

47. If $\lim\limits_{n \to \infty} a_n^{1/n} = \lim\limits_{n \to \infty} \frac{a_{n+1}}{a_n}$ provided limit on RHS exists. This result is known as :

(1) Cauchy 1st Theorem on Limits

(2) Cauchy 2nd Theorem on Limits

(3) Squeeze Principle

(4) Leibnitz' Rule

48. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ is :

(1) 1 (2) 0 (3) e (4) 2

49. Consider the sequence $a_n = \left(1+(-1)^n \frac{1}{n}\right)^n$ then :

(1) $\lim_{n\to\infty} \text{Sup}(a_n) = \lim_{n\to\infty} \text{Inf}(a_n) = 1$

(2) $\lim_{n\to\infty} \text{Sup}(a_n) = \lim_{n\to\infty} \text{Inf}(a_n) = e$

(3) $\lim_{n\to\infty} \text{Sup}(a_n) = \lim_{n\to\infty} \text{Inf}(a_n) = \frac{1}{e}$

(4) $\lim_{n\to\infty} \text{Sup}(a_n) = e, \lim_{n\to\infty} \text{Inf}(a_n) = \frac{1}{e}$

50. If X and Y are standard normal variates with correlation coefficient 'ρ' between them, then the correlation coefficient between X^2 and Y^2 is :

(1) $2\rho - 1$ (2) ρ^2 (3) ρ (4) $\sqrt{\rho}$

51. Let Y_1 , Y_2 , Y_3 be uncorrelated observations with common variance σ^2 and expectations given by $(Y_1) = \beta_1$, $E(Y_2) = \beta_2$ and $E(Y_3) = \beta_1 + \beta_2$ where β_1 and β_2 are unknown parameters. The best linear unbiased estimator of $\beta_1 + \beta_2$ is :

(1) $\frac{1}{2}(Y_1+Y_2+Y_3)$ (2) $Y_1 + Y_2$

(3) $\frac{1}{2}(Y_1+Y_2+2Y_3)$ (4) $\frac{1}{2}(Y_1-Y_2-Y_3)$

52. Under the regulatory conditions, if λ_n is the likelihood ratio, then the asymptotic distribution of $-2\log \lambda_n$ as $n \to \infty$ is :

(1) Normal distribution (2) Chi square distribution

(3) F-distribution (4) T distribution

53. If the pdf of a random variable 'X' is $f(x, \theta) = 1 - \theta^2$, $0 \le x \le \frac{1}{1-\theta^2}$ and if we reject $H_0 : \theta = \frac{1}{2}$ against the alternative $H_1 : \theta = \frac{3}{4}$ whenever $x \ge 1$, then what is the power of the test ?

(1) $\frac{15}{16}$ (2) $\frac{1}{2}$ (3) $\frac{7}{16}$ (4) $\frac{9}{16}$

54. If $X_1, X_2, \ldots\ldots X_n$ is a random sample form poisson distribution with parameter 'λ', then the maximum likelihood estimator of $\log \lambda$ is given by :

(1) $e^{\left(\sum_{i=1}^{n} \frac{X_i}{n}\right)}$

(2) $\log\left(\sum_{i=1}^{n} \frac{X_i}{n}\right)$

(3) $e^{\left(\sum_{i=1}^{n} X_i\right)}$

(4) $\log\left(\sum_{i=1}^{n} X_i\right)$

55. Which of the following could be used as a test for autocorrelation up to third order ?

(1) Spearmen Correlation test

(2) The Breusch-Goldfrey test

(3) The Durbin Watson test

(4) The Gold-Fled Quandt test

56. The following equation represent a simultaneous equation model :

$$K_1 = \alpha_1 K_2 + \beta_1 Z_1 + u_1$$

$$K_2 = \alpha_2 K_1 + \beta_2 Z_2 + u_2$$

OLS will suffer from simultaneous bias if :

(1) u_1 is correlated with Z_1

(2) Z_1 is correlated with Z_2

(3) K_2 is correlated with u_1

(4) K_1 is correlated with u_1

57. Having known the last census population 'P_0' and growth rate 'r', the population after 'n' years based on compound interest formula will be :

(1) $\hat{P}_t = P_0(1+r)^n$

(2) $\hat{P}_t = P_0(1+n)^r$

(3) $\hat{P}_t = P_0/(1+r)^n$

(4) $\hat{P}_t = P_0/(1+n)^r$

58. Vital rates are customarily expressed as :

(1) Percentages (2) Per Thousand

(3) Per Million (4) Per Trillion

59. Age - Specific mortality rates fail to reveal :

(1) Mortality conditions (2) Age-distribution of persons

(3) Sex Ratio (4) All of the above

60. The relationship between NRR and GRR is :

(1) NRR and GRR are usually equal

(2) NRR can never exceed GRR

(3) NRR is generally greater than GRR

(4) None of the above

61. For a standard $n \times n$ Latin Square, how many different Latin squares can be obtained with the same standard ?

(1) $n!\,(n-2)!$ (2) $(n-1)!\,(n-2)!$

(3) $n!\,(n-1)!$ (4) $n!\,(n+1)!$

62. The condition for the time reversal test to hold good with usual notations is :

(1) $P_{01} \times P_{10} = 1$ (2) $P_{10} \times P_{01} = 0$

(3) $P_{01} / P_{10} = 1$ (4) $P_{01} + P_{10} = 1$

63. Weight in Laspeyre's price index number is known as ....

(1) Quantity during the current year

(2) Quantity in the base year

(3) Price during the current year

(4) Price in the base year

64. In India, the collection of vital statistics started for first time in :

(1) 1720 (2) 1886 (3) 1969 (4) 1946

65. Fisher's ideal formula does not satisfy ....

(1) Time Reversal Test (2) Circular Test

(3) Factor Reversal Test (4) Unit Test

66. If l_x is the number of persons living at the age 'x' and 'L_x' the number of persons living in the mid of 'x' and '$(x+1)$' years, then the relation between l_x and L_x is :

(1) $L_x = \frac{1}{2}(l_x + l_{x+1})$ (2) $L_x = \left(\frac{x}{2} + l_x\right)$

(3) $L_x = l_{x+\frac{1}{2}}$ (4) $L_x = l_{x+\frac{3}{2}}$

67. Fertility rates mainly depend on :

(1) Total female population

(2) Total population

(3) Female population of child bearing age

(4) Number of newly born babies

68. If the quantity demanded of a commodity is unresponsive to change in prices, then the demand of that commodity is :

(1) Perfectly Inelastic (2) Elastic

(3) Unit Elastic (4) Inelastic

69. The elasticity for the demand of the durable goods is :

(1) Zero (2) Equal to unity

(3) Greater than unity (4) Less than Unity

70. Consider the following matrix :

$$P = \begin{bmatrix} 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \\ 1/2 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \end{bmatrix}$$

If P is a stochastic matrix, then which of the following is ***not*** true ?

(1) P is Ergodic (2) P is Regular

(3) P is not Regular (4) Both (1) and (3)

71. Service time in queueing theory is usually assumed to follow :

(1) Poisson Distribution (2) Erlang Distribution

(3) Exponential Distribution (4) Normal Distribution

72. For M|M|1 queueing system, the expected number of customers in systems are :

(1) $L_S = \left(\frac{\lambda}{\mu - \lambda}\right)$ (2) $L_S = \left(\frac{\lambda - \mu}{\lambda}\right)$

(3) $L_S = \left(\frac{\mu}{\mu - \lambda}\right)$ (4) $L_S = \left(\frac{\mu - \lambda}{\lambda}\right)$

73. Which of the following relationships is ***not*** true ?

(1) $W_S = W_q + \frac{1}{\mu}$ (2) $L_S = \lambda W_S$

(3) $L_S = L_q + \frac{1}{\lambda}$ (4) $L_q = \lambda W_q$

74. Maximize $Z = 10x_1 + 25x_2$, subject to $0 \le x_1 \le 3, 0 \le x_2 \le 3, x_1 + x_2 \le 5$

(1) 80 at (3, 2) (2) 75 at (0,3) (3) 30 at (3,0) (4) 95 at (2, 3)

75. Which variable is added for the less than or equal to type of constraint ?

(1) Slack (2) Surplus (3) Artificial (4) Basic

76. The convex combination of two points $\bar{x}_1, \bar{x}_2 \in X$ is referred as :

(1) $(1-\lambda)\bar{x}_1 + \lambda\bar{x}_2, 0 \le \lambda \le 1$

(2) $(1-\lambda)\bar{x}_1 + \lambda\bar{x}_2$, λ is any real number

(3) $\bar{x}_1 + \lambda\bar{x}_2, 0 \le \lambda \le 1$

(4) $\lambda\bar{x}_1 + \lambda\bar{x}_2$, λ is any real number

77. The assignment problem is :

(1) non-linear programming problem

(2) dynamic programming problem

(3) integer linear programming problem

(4) integer non-linear programming problem

78. The order of Convergence of Secant Method is :

(1) 2.4 (2) 2 (3) 1.62 (4) 1

79. If $f(0) = 3, f(1) = 5, f(3) = 21$, then the unique polynomials of degree 2 or less using Newton divided difference interpolation will be :

(1) $2x^2 + 2x + 1$

(2) $2x^2 - 3x + 1$

(3) $2x^2 + 3$

(4) $x^2 + 3x - 2$

80. The third difference of a cubic $\Delta^3 y$ function are :

(1) Constant

(2) Not constant

(3) Variables

(4) None of the above

81. The mean of Binomial distribution $B(n, p)$ is :

(1) np (2) $np - 1$ (3) $np - 2$ (4) $np - 3$

82. Given that $E[X + 4] = 10$ and $E[X + 4]^2 = 116$, then Var$[X]$ is equal to :

(1) 4 (2) 8 (3) 12 (4) 16

83. Let 'X' be a continuous random variable with Probability Density Function

$$f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ -ax + 3a, & 2 \le x \le 3 \\ 0, & x > 3 \end{cases}$$

Then the value of 'a' is given by :

(1) 0.4 (2) 0.5 (3) 0.3 (4) 0.1

84. Let 'X' be a continuous random variable with Probability Density Function

$$f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ -ax + 3a, & 2 \le x \le 3 \\ 0, & x > 3 \end{cases}$$

Then the value of $P(X \le 1.5)$ is given by :

(1) 0.5 (2) 0.81 (3) 0.19 (4) 0.17

85. The moment generating function of Poisson Distribution is :

(1) $e^{\lambda}(e^{t}-1)$ (2) $e^{t}(e^{\lambda}-1)$ (3) $e^{t}-1$ (4) $e^{\lambda}(1-e^{t})$

86. Which of the following is the median of the exponential distribution with parameter λ ?

(1) λ (2) $-\lambda^{-1}$ (3) λ^{-1} (4) λ^{-2}

87. The Quartile Deviation of the normal distribution is :

(1) Q. D. = $\frac{2}{4}\sigma$ (2) Q. D. = $\frac{2}{3}\sigma$ (3) Q. D. = $\frac{2}{5}\sigma$ (4) Q. D. = $\frac{3}{4}\sigma$

88. If A and B are two independent events, then $P(\overline{A} \cap \overline{B})$ is equal to :

(1) $P(\overline{A})P(\overline{B})$ (2) $1-P(A \cup B)$

(3) $[1-P(A)][1-P(B)]$ (4) All of the above

89. If event A and event B has occurred and it is known that $P(B) = 1$, the conditional probability $P(A/B)$ is equal to :

(1) $P(A)$ (2) $P(B)$ (3) One (4) Zero

90. The recurrence formula for geometric distribution is given by :

(1) $p(x+i) = qp(x+i)$ (2) $p(x) = qp(x+i)$

(3) $p(x+i) = qp(x)$ (4) $p(x) = p(x+i)$

91. The correct relationship between A.M, G.M., and H.M. is :

(1) A.M.= G.M.= H.M. (2) G.M. ≥ A.M. ≥ H.M.

(3) H.M. ≥ G.M. ≥ A.M. (4) A.M. ≥ G.M. ≥ H.M

92. Average wages of workers of factory are Rs. 550 per month and the standard deviation of wages is 110. The coefficients of variation is :

(1) 30% (2) 15% (3) 500% (4) 20%

93. If the mode of a frequency distribution is 16 and its mean is 16, then the median of the distribution is :

(1) 0 (2) 16 (3) 32 (4) 8

94. If Quartile deviation of a set of observations is given as 6.4 and the value of first quartile is 5. What is the value of the third quartile ?

(1) 12 (2) 15.3 (3) 17.8 (4) 20.2

95. For a leptokurtic frequency curve, the measures of kurtosis is :

(1) 0 (2) −3 (3) less than 1 (4) greater than 3

96. Standard error of the sample correlation coefficient '*r*' is based on '*n*' paired values is :

(1) $\frac{1+r^2}{\sqrt{n}}$ (2) $\frac{1+r^2}{n}$ (3) $\frac{1-r^2}{\sqrt{n}}$ (4) $\frac{1+r^2}{\sqrt{n-1}}$

97. Given the following set of equations :

$$x_1+4x_2-x_3=3$$
$$5x_1+2x_2+3x_3=4$$

The basic feasible solution involving x_1 and x_2 is :

(1) $\left(\frac{5}{9},\frac{11}{18},0\right)$ (2) $\left(\frac{5}{9},0,0\right)$ (3) $\left(0,\frac{11}{18},0\right)$ (4) $\left(\frac{2}{9},0,\frac{3}{14}\right)$

98. The range of a partial correlation coefficient is :

(1) 0 to ∞ (2) −∞ to ∞ (3) 0 to 1 (4) −1 to 1

99. The term regression was introduced by :

(1) Sir Francis Galton (2) R.A. Fisher

(3) Karl Pearson (4) P.C. Mahalanobis

100. If Regression Equations are :

$$6y = 5x + 90$$
$$15x = 8y + 30$$

And the variance of $x = 4$, then the correlation coefficient between '*x*' and '*y*' is :

(1) 0.45 (2) 0.67 (3) 0.78 (4) 0.88

Answer keys of PHD-EE-2023-24 (STATISTICS) entrance exam dated 22.03.2024				
Q. NO.	A	B	C	D
1	1	3	1	2
2	4	1	4	2
3	2	2	1	3
4	1	2	1	3
5	1	2	3	3
6	3	3	2	1
7	2	3	2	2
8	4	1	1	1
9	1	3	2	1
10	3	2	4	1
11	4	3	3	3
12	4	4	1	1
13	2	4	3	4
14	3	1	4	2
15	4	1	1	1
16	3	2	1	3
17	1	1	3	3
18	4	2	3	2
19	1	2	3	2
20	2	2	1	2
21	1	2	3	3
22	4	2	2	4
23	1	3	4	4
24	1	3	2	1
25	3	3	2	1
26	2	1	3	2
27	2	2	1	1
28	1	1	2	2
29	2	1	4	2
30	4	1	2	2
31	3	1	3	1
32	1	4	4	4
33	4	2	4	1
34	2	1	1	1
35	1	1	1	3
36	3	3	2	2
37	3	2	1	2
38	2	4	2	1
39	2	1	2	2
40	2	3	2	4
41	3	3	4	4
42	1	1	4	2
43	2	3	2	2
44	2	4	3	1
45	2	1	4	3
46	3	1	3	1
47	3	3	1	2
48	1	3	4	3
49	3	3	1	4
50	2	1	2	2

22/03/2024 22/03/2024 22/3/24 22/3/24

Answer keys of PHD-EE-2023-24 (STATISTICS) entrance exam dated 22.03.2024				
Q. NO.	A	B	C	D
51	3	4	3	3
52	1	2	1	2
53	3	2	4	4
54	4	1	2	2
55	1	3	1	2
56	1	1	3	3
57	3	2	3	1
58	3	3	2	2
59	3	4	2	4
60	1	2	2	2
61	4	1	1	3
62	2	4	4	1
63	2	1	2	2
64	1	1	1	2
65	3	3	1	2
66	1	2	3	3
67	2	2	2	3
68	3	1	4	1
69	4	2	1	3
70	2	4	3	2
71	3	4	3	3
72	4	4	1	1
73	4	2	2	3
74	1	3	2	4
75	1	4	2	1
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79	2	1	3	3
80	2	2	2	1
81	3	3	2	1
82	2	2	2	4
83	4	4	3	2
84	2	2	3	1
85	2	2	3	1
86	3	3	1	3
87	1	1	2	2
88	2	2	1	4
89	4	4	1	1
90	2	2	1	3
91	2	3	4	4
92	2	1	2	4
93	3	4	2	2
94	3	2	1	3
95	3	1	3	4
96	1	3	1	3
97	2	3	2	1
98	1	2	3	4
99	1	2	4	1
100	1	2	2	2

Hamelal 22/03/2024 Amalik 22/03/2024 22/3/24 Gagan 22/3/24